MIMO1: Single symbol over flat fading CSIT/CSIR

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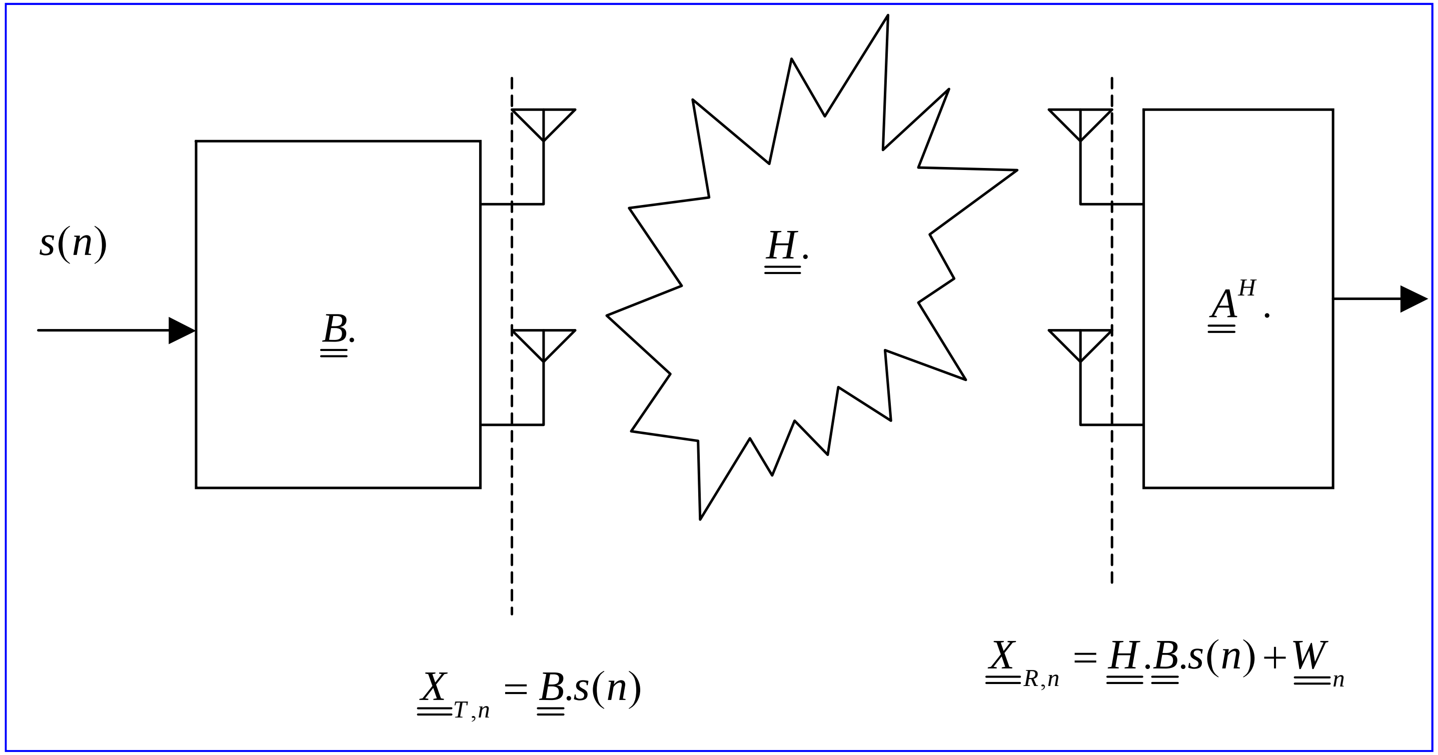
The Scenario



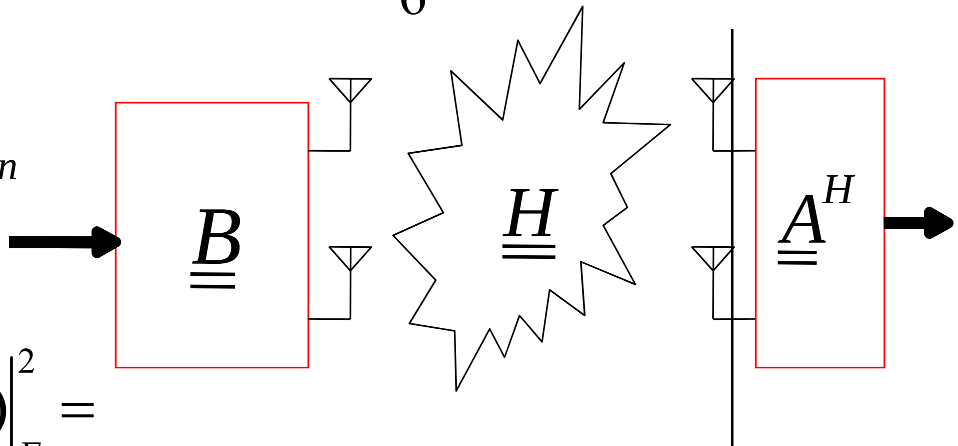
Flat Fading, CSIT and CSIR, Single Symbol

The ML Receiver

$$\underline{\underline{X}}_{T,n} = \underline{\underline{B}}.s(n)$$

$$E_T = trace\left(\underline{\underline{B}}.\underline{\underline{B}}^H\right).E\left(\left|s(n)\right|^2\right) = d^2.\frac{2^{n_s}-1}{6}.trace\left(\underline{\underline{B}}.\underline{\underline{B}}^H\right)$$

$$\underline{\underline{X}}_{R,n} = \underline{\underline{H}}.\underline{\underline{B}}.s(n) + \underline{\underline{W}}_n$$



The likelihood

$$\Lambda\left(s(n)\right) = -\left|\underline{\underline{X}}_{R,n} - \underline{\underline{H}}.\underline{\underline{B}}.s(n)\right|_F^2 =$$

$$= -Traza\left[\left(\underline{\underline{X}}_{R,n} - \underline{\underline{H}}.\underline{\underline{B}}.s(n)\right).\left(\underline{\underline{X}}_{R,n} - \underline{\underline{H}}.\underline{\underline{B}}.s(n)\right)^H\right] =$$

$$\rightarrow 2.\mathrm{Re}\left[s(n)^*.Traza\left(\underline{\underline{B}}^H.\underline{\underline{H}}^H.\underline{\underline{X}}_{R,n}\right)\right] - \left|s(n)\right|^2.Traza\left[\underline{\underline{B}}^H.\underline{\underline{H}}^H.\underline{\underline{H}}.\underline{\underline{B}}\right]$$

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ML Receiver

$$\underline{\underline{A}} = \underline{\underline{H}}.\underline{\underline{B}}$$ Optimum receiver

$$when \quad \Lambda(s(n)) > \Lambda(s(m)) \quad \forall m \neq n \quad decide \quad s(n)$$

After some manipulations and with $\tilde{s}(n) = s(n) - s(m)$

$$Traza\left[\left(\underline{\underline{B}}^H.\underline{\underline{H}}^H.\underline{\underline{H}}.\underline{\underline{B}}\right).\left|\tilde{s}(n)\right|^2\right] > 2.\mathrm{Re}\left[\tilde{s}(n).Traza\left(\underline{\underline{W}}_n^H\,\underline{\underline{H}}.\underline{\underline{B}}\right)\right]$$

The global Tx-H matrix

define

$$\gamma = d^2.Trace\left(\underline{\underline{H}}^H.\underline{\underline{R}}_0^{-1}.\underline{\underline{H}}.\underline{\underline{B}}.\underline{\underline{B}}^H\right) = d^2.Trace\left[\underline{\underline{R}}_H.\left(\underline{\underline{B}}.\underline{\underline{B}}^H\right)\right]$$

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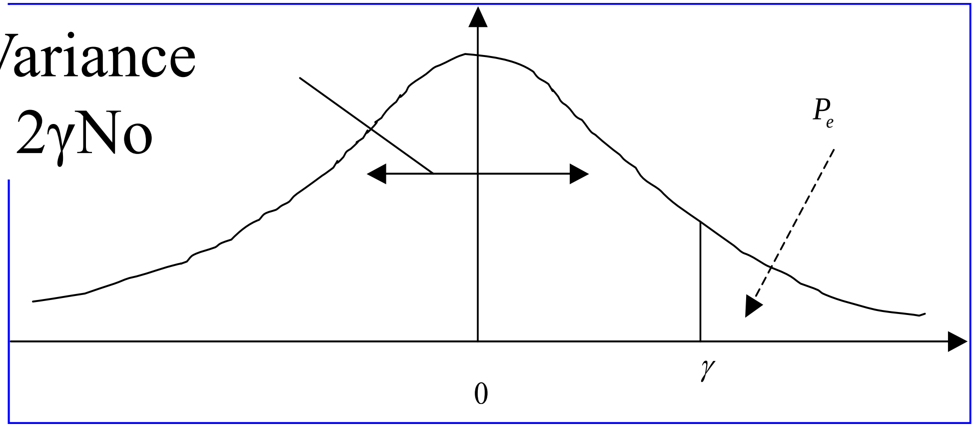
The Probability of Error

$$Traza\left[\left(\underline{\underline{B}}^{H}.\underline{\underline{H}}^{H}.\underline{\underline{H}}.\underline{\underline{B}}\right).\left|\tilde{s}(n)\right|^{2}\right] > 2.\mathrm{Re}\left[\tilde{s}(n).Traza\left(\underline{\underline{W}}_{n}^{H}\,\underline{\underline{H}}.\underline{\underline{B}}\right)\right]$$



Error only to the nearest neighborhood

$$\Pr(s(n)\rightarrow s(m)) = P_e = Q\left(\sqrt{\frac{\gamma}{2.N_0}}\right)$$

Tx Optimization

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$$d^2 trace\left(\underline{\underline{R}}_H . \underline{\underline{B}} . \underline{\underline{B}}^H\right)\Big|_{MAX}$$

$$s.t. \quad E_s . trace\left(\underline{\underline{B}} . \underline{\underline{B}}^H\right) = E_T$$

Max. Quality with a given power budget

The channel matrix

$$\underline{\underline{R}}_H = \underline{\underline{H}}^H \underline{\underline{R}}_0^{-1} . \underline{\underline{H}}$$

The Tx matrix and constraint

$$\underline{\underline{Q}} = (\underline{\underline{B}} . \underline{\underline{B}}^H) . E_s = (\underline{\underline{B}} . \underline{\underline{B}}^H) . d^2 . \frac{2^{n_b} - 1}{6}$$

$$Trace\left(\underline{\underline{Q}}\right) = d^2 . \frac{2^{n_b} - 1}{6} . Trace(\underline{\underline{B}} . \underline{\underline{B}}^H) \leq E_T$$

Full CSIT Design

$$d^2 trace\left(\underline{\underline{R}}_H . \underline{\underline{B}} . \underline{\underline{B}}^H\right)\Big|_{MAX}$$

$$s.t. \quad E_s . trace\left(\underline{\underline{B}} . \underline{\underline{B}}^H\right) = E_T$$

Since, for definite positive matrixes, we have:

$$Trace\left(\underline{\underline{F}} . \underline{\underline{G}}\right) \leq \lambda_{\max}\left(\underline{\underline{F}}\right) . Trace\left(\underline{\underline{G}}\right)$$

Equal when:

$$\underline{\underline{B}} = E_T . \underline{e}_{\max} . \underline{cs}^H \qquad \forall \underline{cs} \text{ unit norm}$$

Full CSIT Architecture and Performance

$$P_e = Q\left(\sqrt{\frac{E_T}{N_0}.\lambda_{MAX}\left(\underline{\underline{R}}_H\right).\frac{3}{2^{n_s}-1}}\right)$$

$$\underline{\underline{H}} = \underline{\underline{V}}_H . \underline{\underline{L}} . \underline{\underline{U}}^H$$

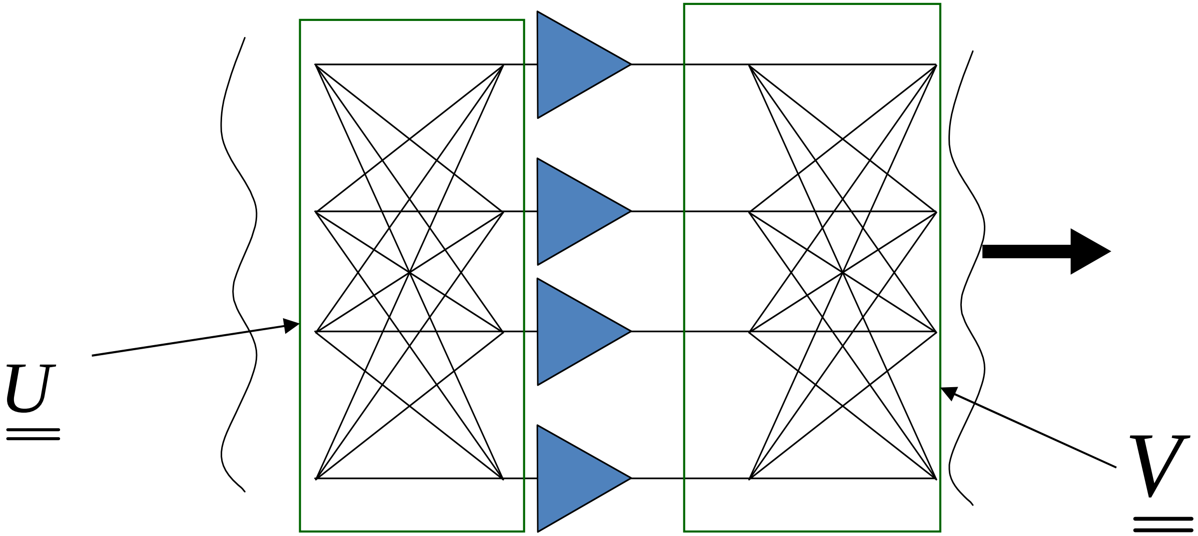
Transmiter

Receiver

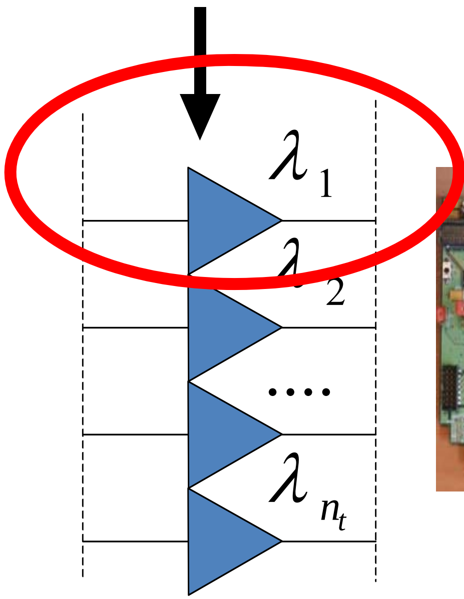
Channel gain

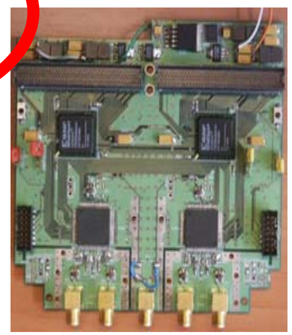
$$\underline{\underline{A}} = \underline{\underline{H}}.\underline{\underline{B}} = \left(\sum_{r=1}^{\min(n_T,n_R)} \lambda_r . \underline{v}_r . \underline{u}_r^H\right).E_T.\underline{u}_{\max} = \lambda_{\max}.E_T.\underline{v}_{\max}$$

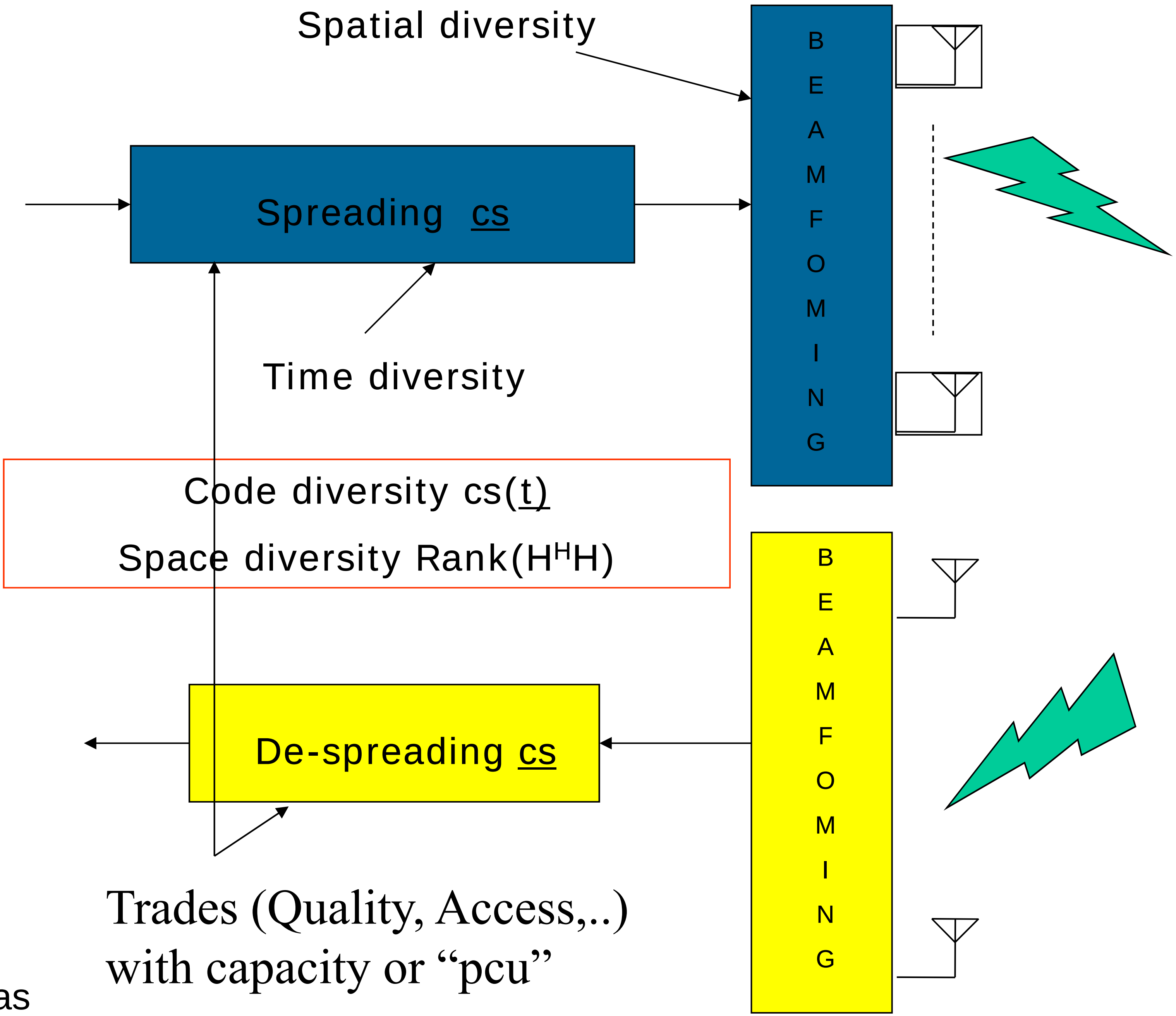
Intuitive Explanation



Just use the best channel for all the available Tx power







Trades (Quality, Access,..) with capacity or "pcu"

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CSIR Only (Define a Game)

Define PLAYERS

- TX Engineer
- Channel

Define payoffs

- Minimize P_E
- Destroy Rx Energy

Define strategy of the game

- Channel plays first, Tx engineer plays second. Only one pass

Constrains/player

- E_T
- $Trace\left(\underline{\underline{R}}_H\right) \geq \rho$

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CSIR Only

$$\max_{\underline{\underline{Q}}}\left(\min_{\underline{\underline{R}}_h}\left(\underline{\underline{R}}_H.\underline{\underline{Q}}\right)\right)$$

One play game

$$\underline{\underline{R}}_H = \underline{\underline{H}}^H.\underline{\underline{R}}_0^{-1}.\underline{\underline{H}} \qquad \underline{\underline{Q}} = \underline{\underline{B}}.\underline{\underline{B}}^H$$

$$Traza\left(\underline{\underline{R}}_H.\underline{\underline{B}}.\underline{\underline{B}}^H\right) \geq \lambda_{\min}\left(\underline{\underline{B}}.\underline{\underline{B}}^H\right).Traza\left(\underline{\underline{R}}_H\right)$$

Channel plays

Tx Eng. plays as maximize the minimum eigenvalue when the trace is constrained

$$\underline{\underline{B}} = \left(\frac{E_T}{n_T}\right)^{1/2}.\underline{\underline{\Pi}} \qquad with \quad \underline{\underline{\Pi}}.\underline{\underline{\Pi}}^H = \underline{\underline{I}}_{n_T}$$

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The CSIR Performance

$$\underline{\underline{B}} = \underline{\underline{\Pi}}.\sqrt{\frac{k_2}{n_T}} \quad y \quad \gamma = 4E_s.Traza\left(\underline{\underline{R}}_H.\underline{\underline{B}}.\underline{\underline{B}}^H\right) = \frac{4.E_s.k_2}{n_T}.Traza\left(\underline{\underline{R}}_H\right)$$

$$P_e^{NO-CSI} = Q\left(\sqrt{\frac{E_T}{N_0}.\left(\frac{Traza\left(\underline{\underline{R}}_H\right)}{n_T}\right).\left(\frac{3}{\left(2^{n_S}-1\right)}\right)}\right)$$

We pass from the maximum eigenvalue gain to the arithmetic mean of the eigenvalues.

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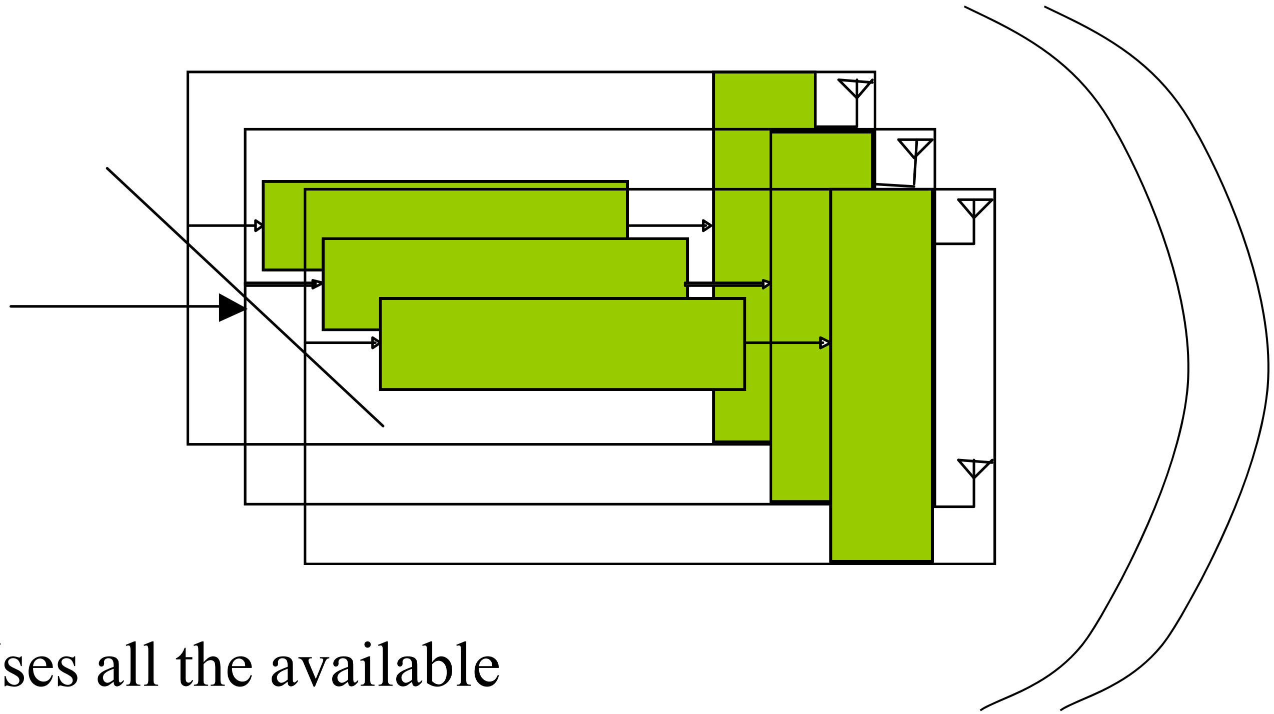
The CSIR only Strategy



Uses all the available eigenmodes

UPA

Uniform Power Allocation

PCU and Increase Rate

Since $\underline{\underline{B}}$ Is unitary, it has to be full rank, i.e. Size n_T by n_T (at least)

One symbol for n_T channel uses → Rate $1/n_T$

It is possible to increase this rate??

SPACE-TIME BLOCK CODES

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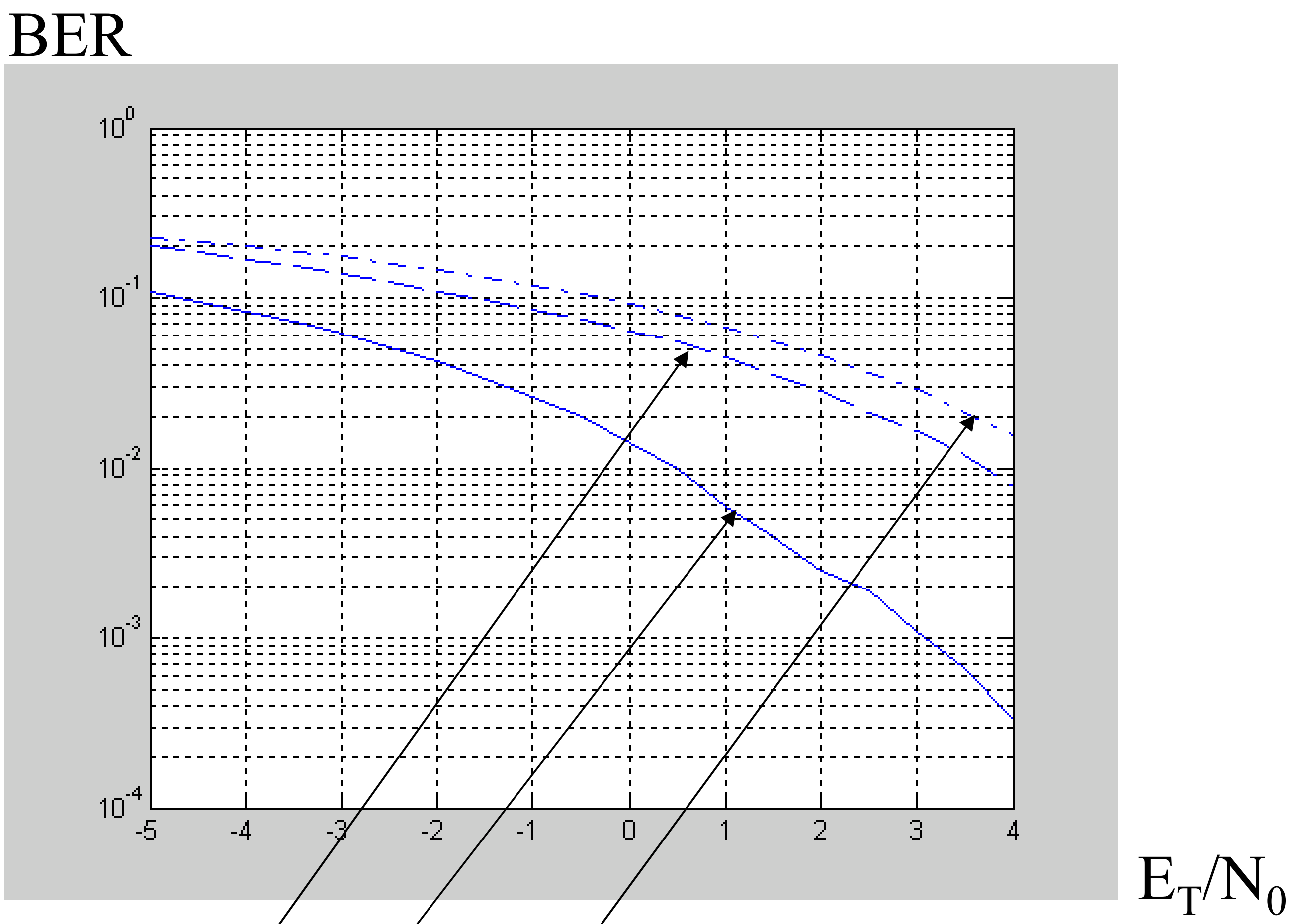

BER versus SNR for a MIMO system without CSIT. De abajo a arriba: MIMO(4,4) six streams and four PCUs (r=3/4); MIMO(3,3); MIMO(2,2) four streams and 2 PCUs (r=1).

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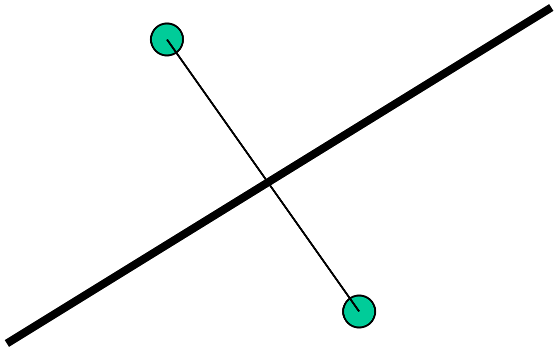
Instantaneous Detection

$$\Lambda\left(s(n)\right) = 2.\mathrm{Re}\left[s(n)^{*}.Traza\left(\underline{\underline{B}}^{H}.\underline{\underline{H}}^{H}.\underline{\underline{X}}_{R,n}\right)\right] - \left|s(n)\right|^{2}.Traza\left[\underline{\underline{B}}^{H}.\underline{\underline{H}}^{H}.\underline{\underline{H}}.\underline{\underline{B}}\right]$$



The nearest neighborhood

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Constant modulus constellation (BPSK, QPSK, M-PSK

$$\mathrm{Re}\left[Traza\left(\underline{\underline{B}}^{H}.\underline{\underline{H}}^{H}.\underline{\underline{X}}_{R,n}\right)\right] \underset{<}{\overset{>}{}} 0$$

$$\mathrm{Im}\left[Traza\left(\underline{\underline{B}}^{H}.\underline{\underline{H}}^{H}.\underline{\underline{X}}_{R,n}\right)\right] \underset{<}{\overset{>}{}} 0$$

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The maximum likelihood detector:

$$\left(\underline{\underline{X}}_{Rn} - \underline{\underline{H}}.\underline{\underline{B}}s(n)\right)^{H} \underline{\underline{R}}_{0}^{-1} \left(\underline{\underline{X}}_{Rn} - \underline{\underline{H}}.\underline{\underline{B}}s(n)\right)$$

with $\underline{\underline{R}}_{0}^{-1} = \underline{\underline{R}}_{0}^{-1/2}.\left(\underline{\underline{\Phi}}.\underline{\underline{\Phi}}^{H}\right).\underline{\underline{R}}_{0}^{-1/2}$

$$\Lambda\left(s(n)\right) = \left|\underline{\underline{\Phi}}^{H}.\underline{\underline{R}}_{0}^{-1/2}.\underline{\underline{X}}_{Rn} - \underline{\underline{\Phi}}^{H}.\underline{\underline{R}}_{0}^{-1/2}.\underline{\underline{H}}.\underline{\underline{B}}.s(n)\right|_{F}$$

$$\underline{\underline{A}}^{H}$$

With the previous design:

$$\Lambda\left(s(n)\right) = \left|\underline{v}_{\max}^{H}.\underline{\underline{R}}_{0}^{-1/2}.\underline{\underline{X}}_{Rn} - \lambda_{\max}\left(\underline{\underline{R}}_{H}\right)s(n)\right|^{2}$$

Enabling the symbol by symbol detection