

innovating communications

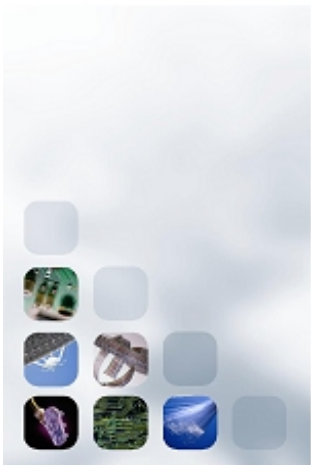
## The Centre Tecnològic de Telecomunicacions de Catalunya

*A gateway to advanced communication technologies*

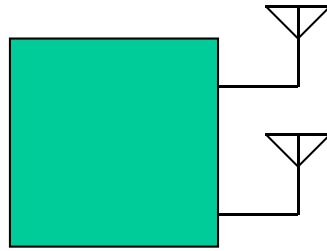
**MIMO-MAC and MIMO-BC**

Miguel Ángel Lagunas

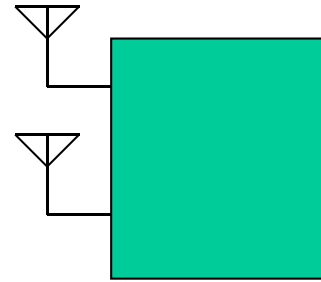
[m.a.lagunas@cttc.es](mailto:m.a.lagunas@cttc.es)



## Full Collaborative



Sum-rate



$$r_1 + r_2 = 2 \log \left[ \frac{E_T}{2} \cdot \Delta^{0.5} + \frac{\text{tr}(R_H)}{2 \cdot \Delta^{0.5}} \right]$$

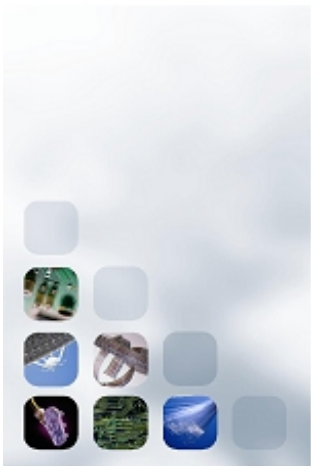
where

$$\Delta \rightarrow \text{determinant} = |\underline{h}_1|^2 \cdot |\underline{h}_2|^2 - |h_{12}|^2$$

$$R_H = \begin{pmatrix} |\underline{h}_1|^2 & h_{12} \\ h_{12}^* & |\underline{h}_2|^2 \end{pmatrix}$$

$$\lambda_{HAR} \rightarrow \text{harmonic mean} = \frac{2 \cdot \Delta}{\text{tr}(R_H)}$$

3

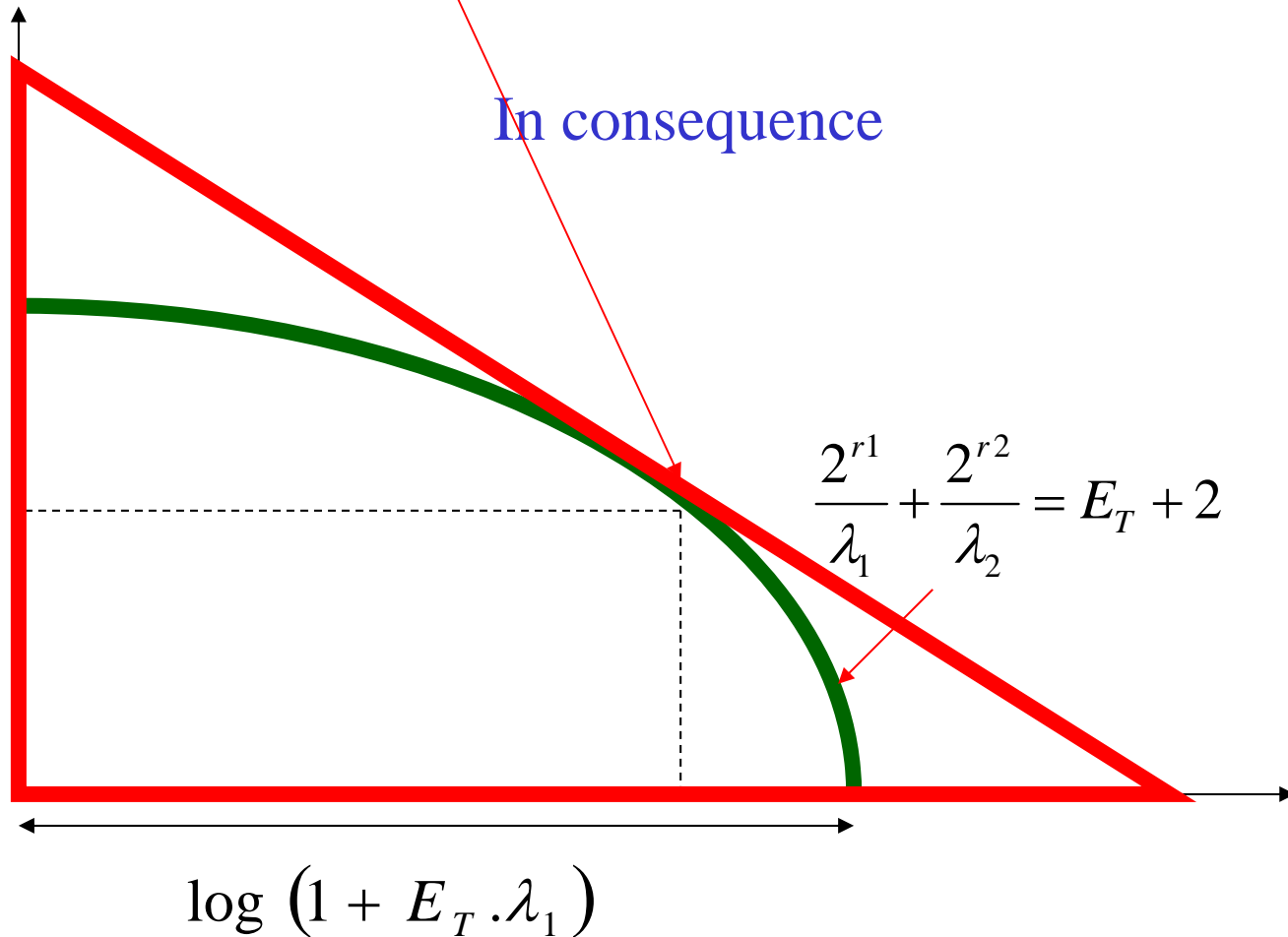


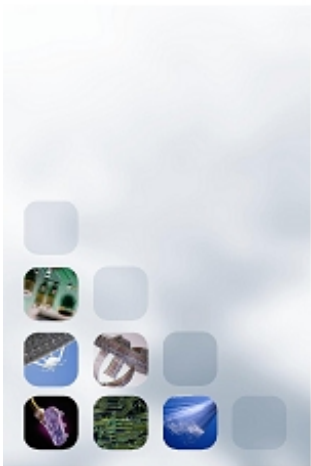
$$r_1 = \log(1 + z_1 \cdot \lambda_1) = \log(\mu \cdot \lambda_1)$$

$$r_2 = \log(1 + z_2 \cdot \lambda_2) = \log(\mu \cdot \lambda_2)$$

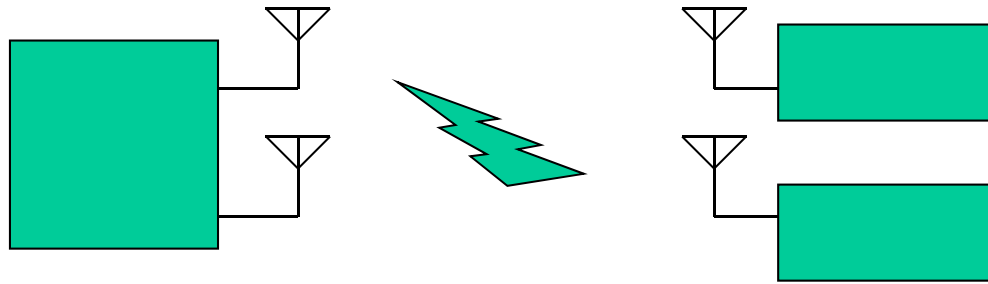
$$\mu = \frac{E_T}{2} + \frac{1}{\lambda_{HARM}}$$

In consequence





## MIMO\_MAC



The Tx cannot diagonalize the channel

The capacity  
bounds are set  
independently  
for every  
transmitter

$$r_1 = \log\left(1 + |h_1|^2 E_1\right)$$

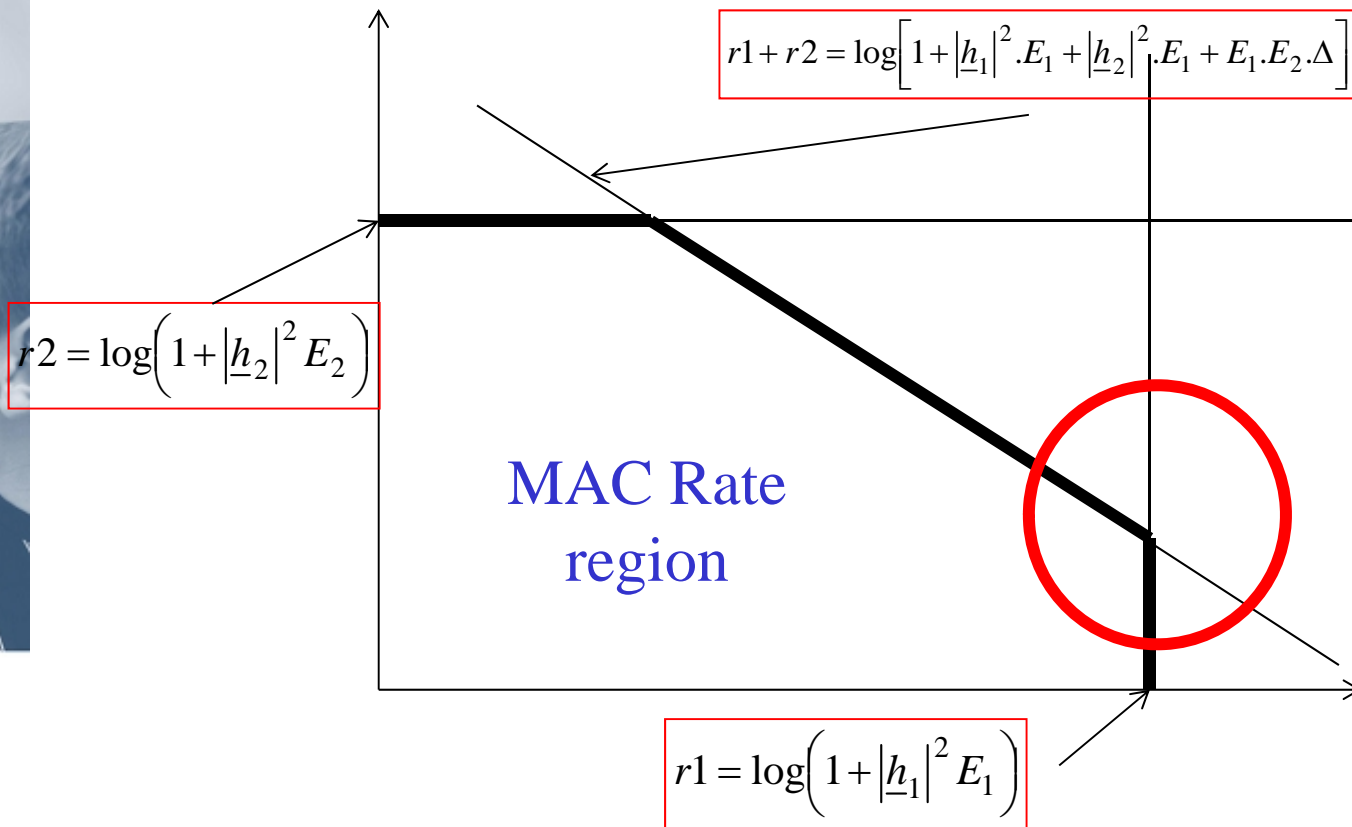
$$r_2 = \log\left(1 + |h_2|^2 E_2\right)$$

As MIMO it is a bound on the sum-rate

$$r_1 + r_2 = \log \left[ \det \left( I + \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} \begin{pmatrix} |h_1|^2 & h_{12} \\ h_{21} & |h_2|^2 \end{pmatrix} \right) \right] =$$

$$= \log \left[ 1 + |h_1|^2 \cdot E_1 + |h_2|^2 \cdot E_1 + E_1 \cdot E_2 \cdot \Delta \right]$$

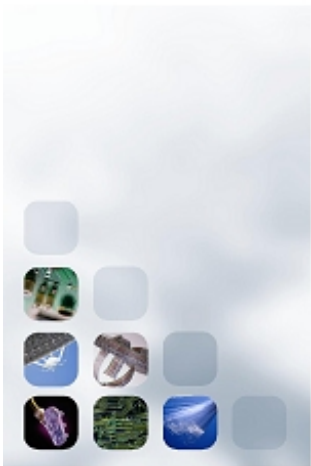
5



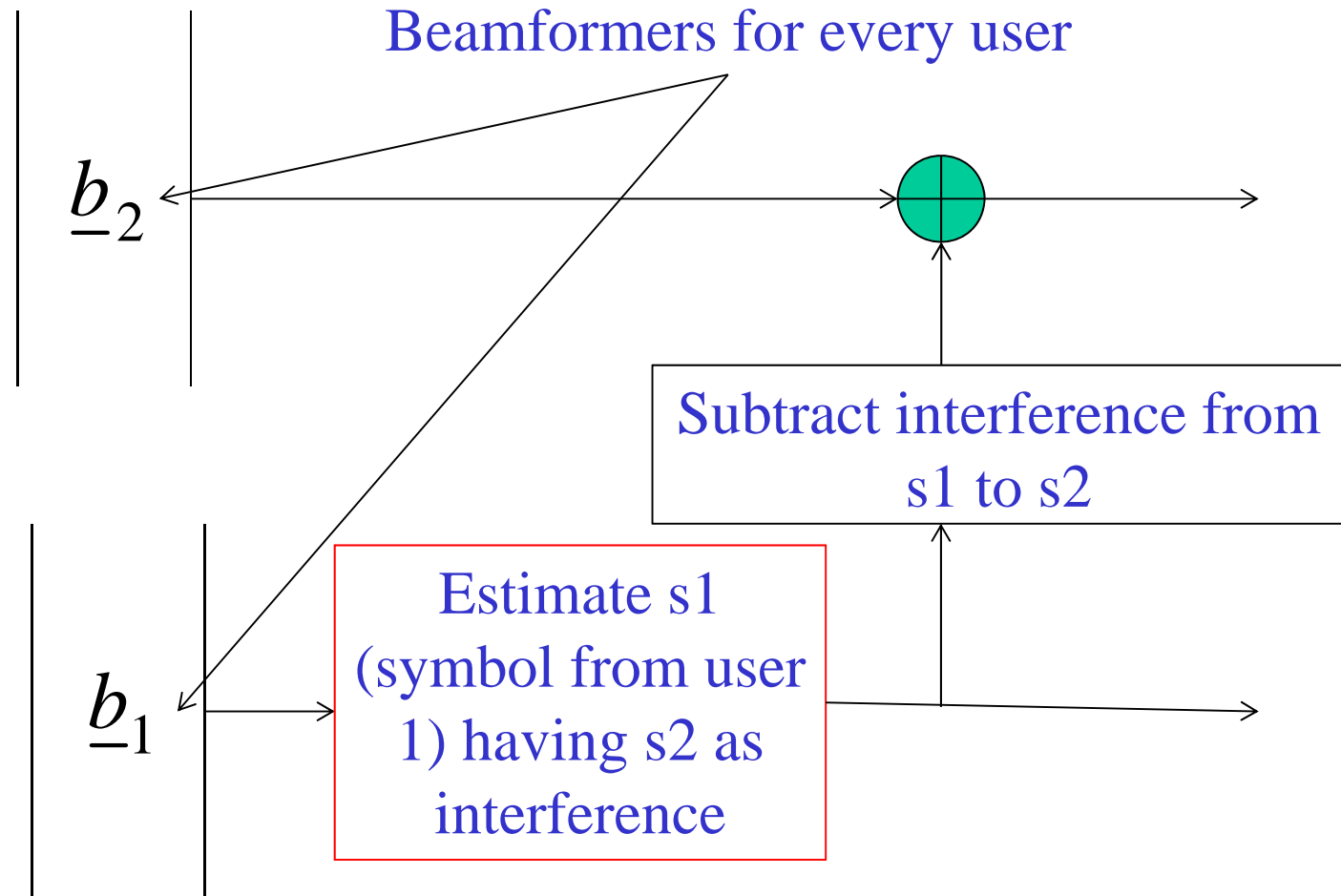
Decoding one user without interference implies that  
(solve for the cross point of the two lines):

$$r_1 B = \log(1 + |h_1|^2 E_1)$$

$$r_2 B = \log\left(1 + \frac{|h_2|^2 E_2}{1 + |h_1|^2 E_1} + \frac{E_1 E_2 \Delta}{1 + |h_1|^2 E_1}\right)$$



## ***Guideline: Estimate the strongest symbol and subtract interference.***



## Zero-Forcing Beamformer

$$\underline{\underline{B}} = \left( \left[ \underline{\underline{I}} - \frac{\underline{h}_2 \underline{h}_2^H}{|\underline{h}_2|^2} \right] \frac{\underline{h}_1}{(1-\phi)^{1/2} |\underline{h}_1|} \quad \left[ \underline{\underline{I}} - \frac{\underline{h}_1 \underline{h}_1^H}{|\underline{h}_1|^2} \right] \frac{\underline{h}_2}{(1-\phi)^{1/2} |\underline{h}_2|} \right)$$

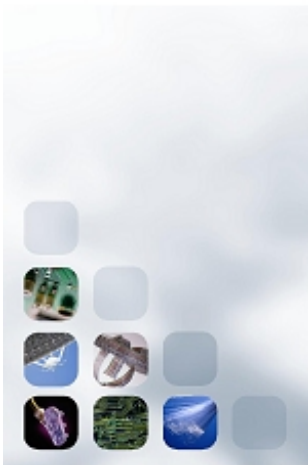
$$\underline{\underline{B}}^H \begin{bmatrix} \underline{h}_1 & \underline{h}_2 \end{bmatrix} = \begin{pmatrix} (1-\phi)^{1/2} |\underline{h}_1| & 0 \\ 0 & (1-\phi)^{1/2} |\underline{h}_2| \end{pmatrix}$$

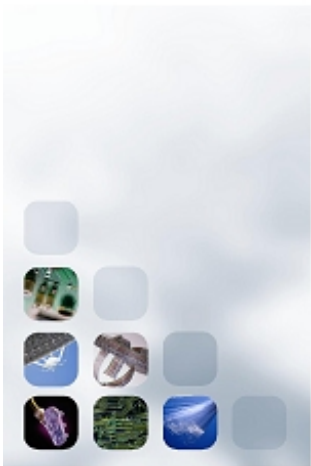
where  $\phi = \frac{|\underline{h}_{12}|^2}{|\underline{h}_1|^2 |\underline{h}_2|^2}$

In consequence, both sum-rate and single rates experience losses

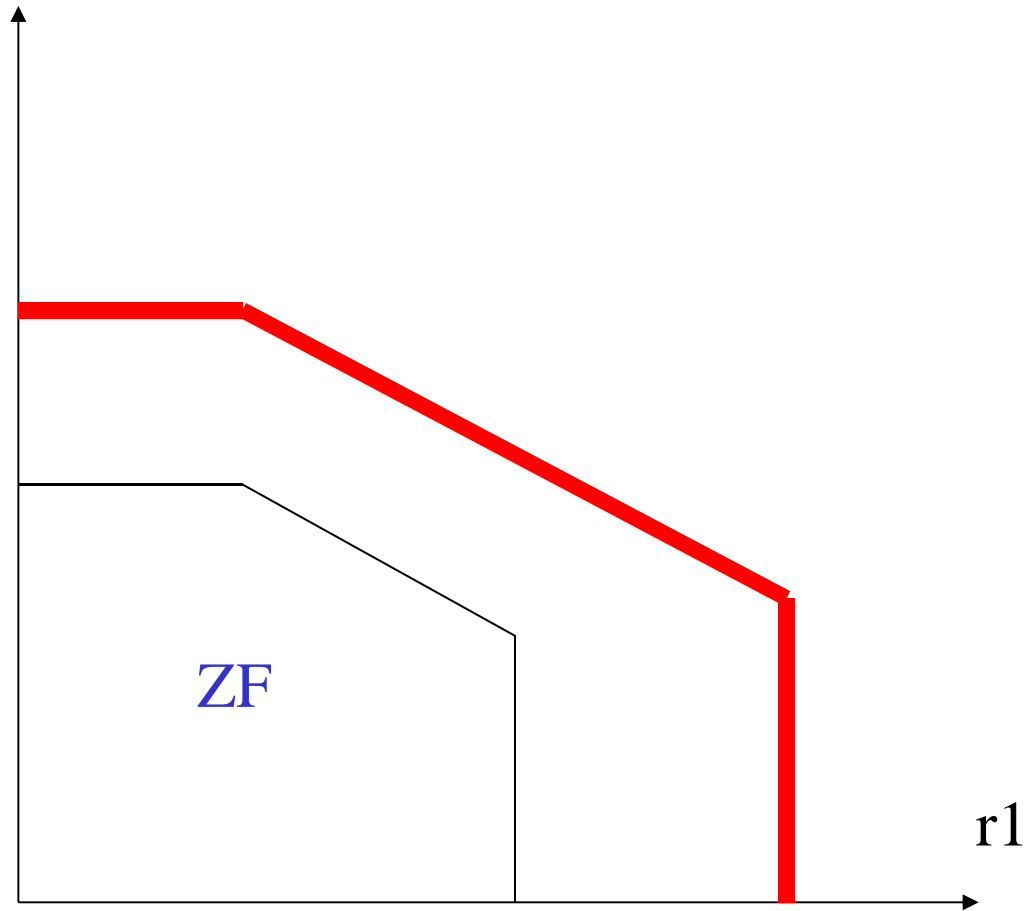
$$r_1 = \log \left[ 1 + (1-\phi) |\underline{h}_1|^2 E_1 \right]$$

$$r_2 = \log \left[ 1 + (1-\phi) |\underline{h}_2|^2 E_2 \right]$$





## ZF Region in the MAC Region





## QR Decomposition and SIC

$$\underline{\underline{B}} = \begin{pmatrix} \frac{\underline{h}_1}{|\underline{h}_1|} & \left[ I - \frac{\underline{h}_1 \underline{h}_1^H}{|\underline{h}_1|^2} \right] \frac{\underline{h}_2}{(1-\phi)^{1/2} |\underline{h}_2|} \end{pmatrix}$$

$$\underline{\underline{B}}^H \begin{bmatrix} \underline{h}_1 & \underline{h}_2 \end{bmatrix} = \begin{pmatrix} |\underline{h}_1| & h_{12} \\ 0 & (1-\phi)^{1/2} |\underline{h}_2| \end{pmatrix}$$

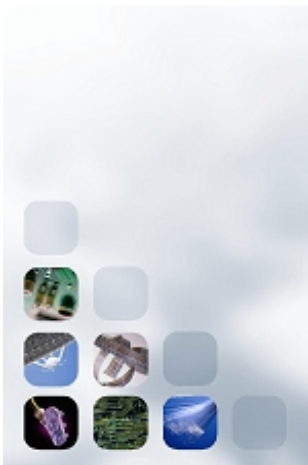
We get s2 free of interference

Does not achieves the sum rate but r1 is detected at full-rate

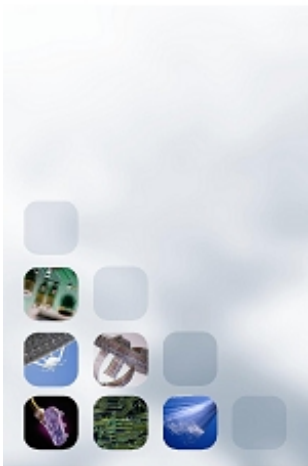
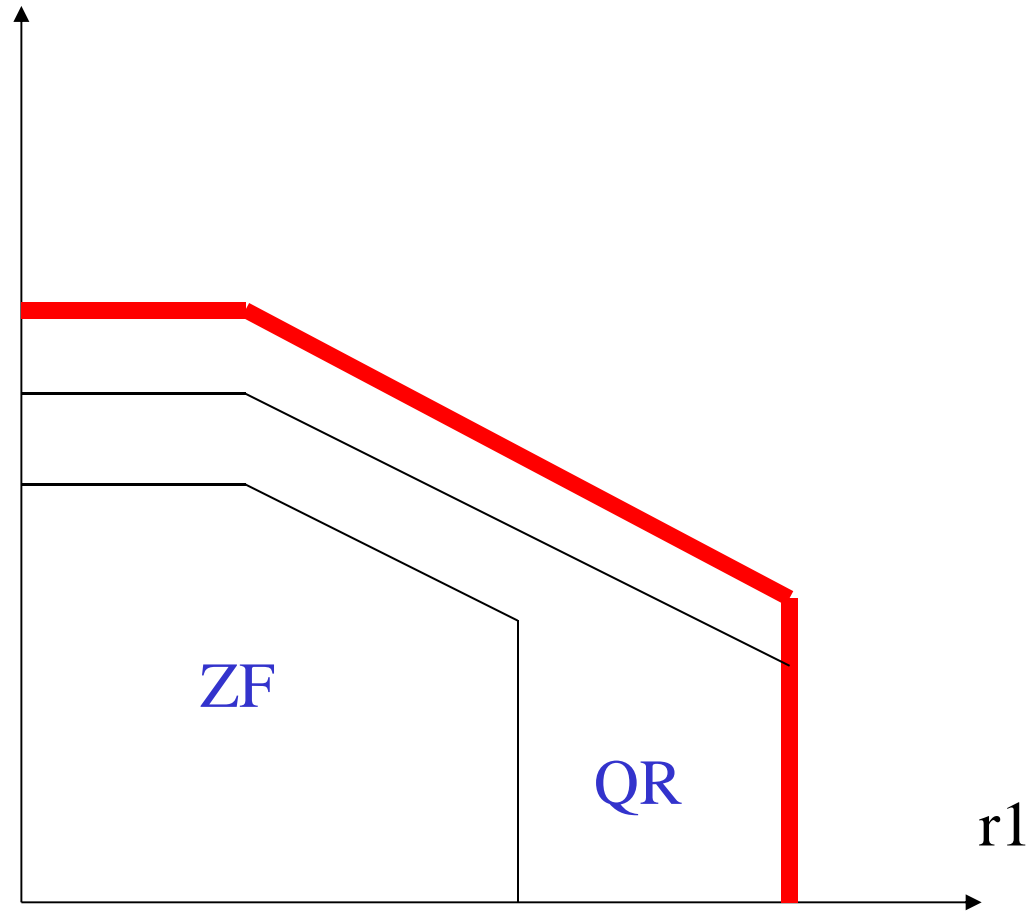
Thus, we can detect s2 and then subtract the interference to s1

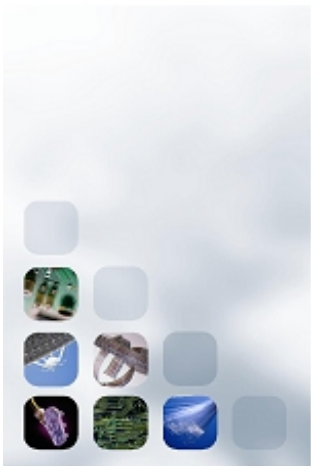
$$r_1 = \log \left[ 1 + |\underline{h}_1|^2 E_1 \right]$$

$$r_2 = \log \left[ 1 + (1-\phi) |\underline{h}_2|^2 E_2 \right]$$



# QR Region in the MAC Region





## SIC: A DSP Formulation

From the likelihood, we can define the following error

$$\underline{R}_0^{-0.5} \left( \underline{X}_{Rn} - \underline{H} \underline{P}^{0.5} \underline{s}_n \right) \Rightarrow |\underline{\varepsilon}_n|^2 = \left| \underline{R}_0^{-0.5} \underline{X}_{Rn} - \underline{R}_0^{-0.5} \underline{H} \underline{P}^{0.5} \underline{s}_n \right|^2$$

Now, using the QR decomposition for the global channel

$$\underline{R}_0^{-0.5} \underline{H} = \underline{Q} \underline{R}$$

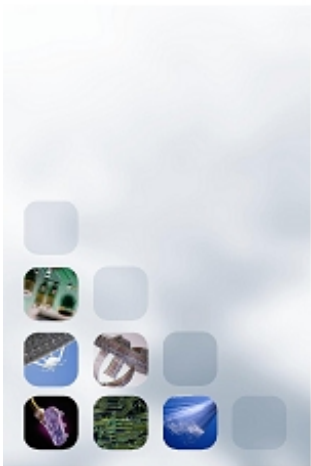
Then, using a beamforming matrix  $\underline{B}$ , at the receiver equal to  $\underline{Q}$ , we have:

$$\underline{B} = \underline{Q} \left( \text{diag}(\text{diag}(\underline{R})) \right)^{-1} \quad \underline{\varepsilon}_n = \underline{Y}_n - \underline{R}_a \underline{s}_n \quad \text{where} \quad \underline{Y}_n = \underline{B}^H \underline{X}_{Rn}$$

Now.....

$$\text{with} \quad \underline{R}^H = \text{diag} \left( \text{diag} \left( \underline{R}^H \right) \right) \underline{R}_a$$

being  $\text{diag} \left( \text{diag} \left( \underline{R}_a \right) \right)$  is the identity matrix



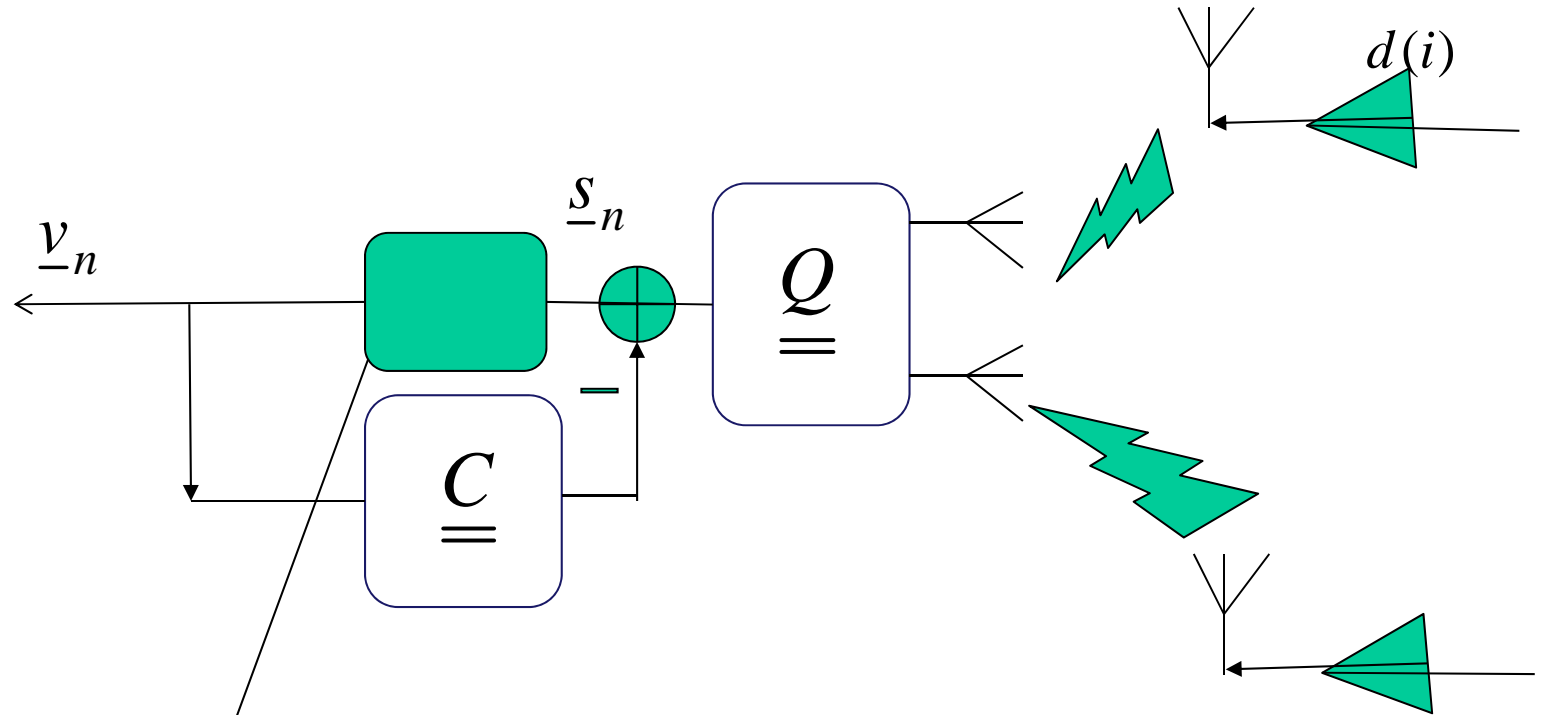
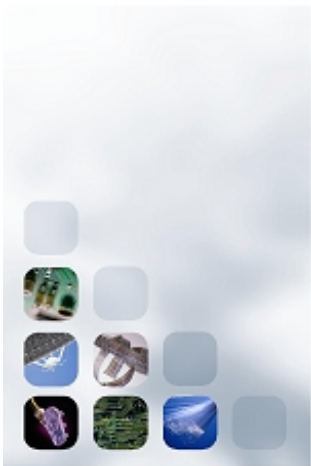
We have

$$\underline{R}_a \underline{s}_n = (\underline{I} + \underline{C}) \underline{s}_n$$

$$\underline{\varepsilon}_n + \underline{s}_n = \underline{Y}_n - \underline{C} \underline{s}_n$$

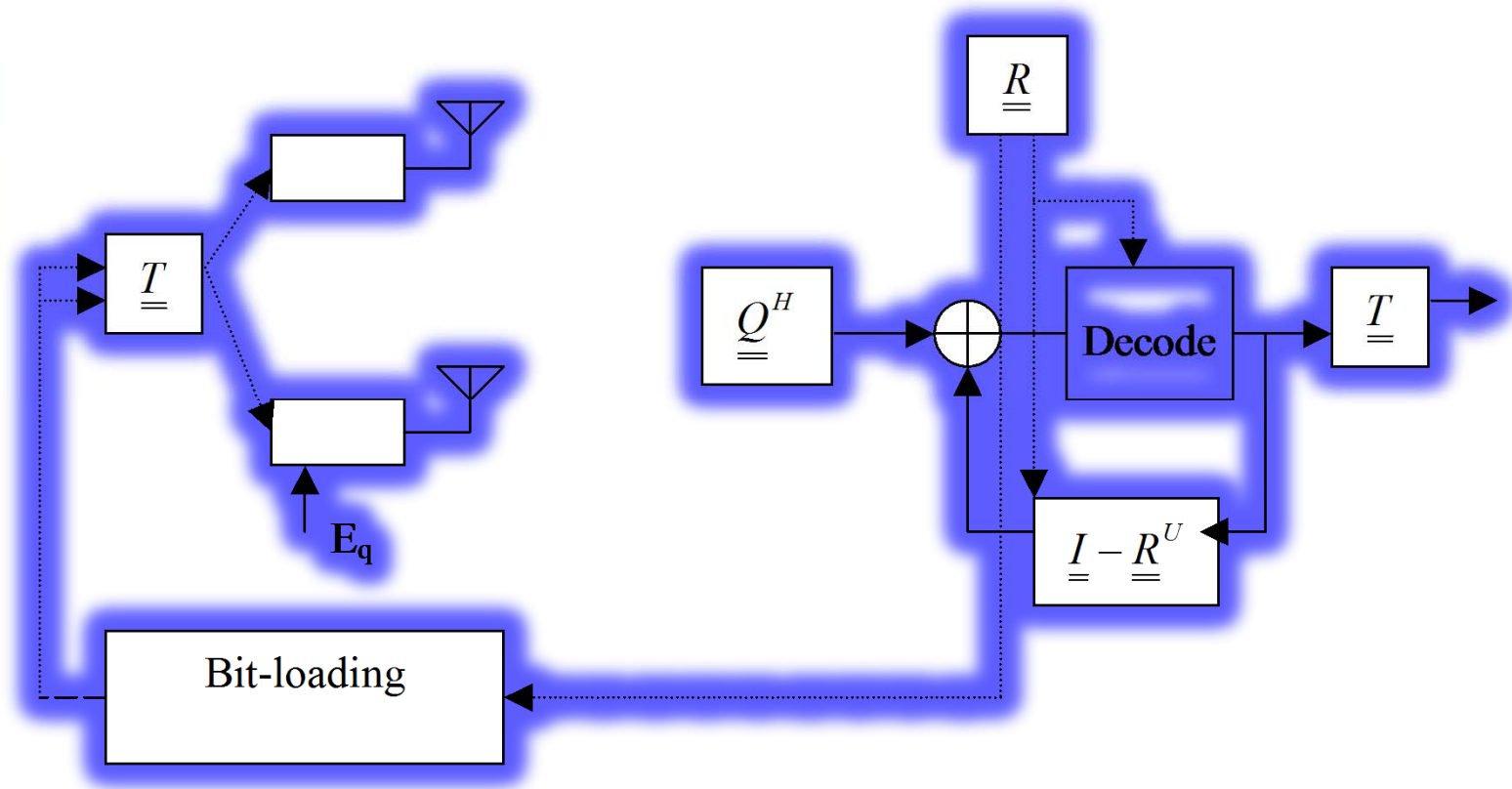
Get an estimate by  
thresholding in  
accordance to the  
given constellation

!!! C is strictly lower  
triangular!!!



Multi-stream  
constellation  
thresholding

# The labeling problem:



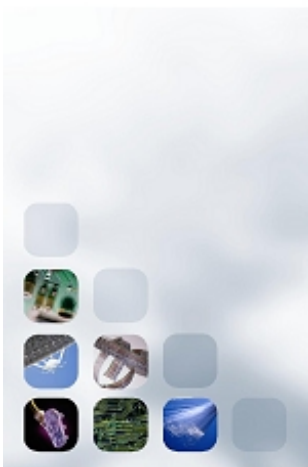
## MSE Beamforming and SIC

$$\underline{\underline{B}} = \left( \begin{array}{c|c} \frac{\underline{h}_1}{|\underline{h}_1|} & \left[ I - \frac{\underline{h}_1 \underline{h}_1^H E_1}{1 + |\underline{h}_1|^2 E_1} \right] \frac{\underline{h}_2}{\alpha} \end{array} \right)$$

where

$$\alpha^2 = \frac{\Delta E_1 + |\underline{h}_2|^2}{1 + |\underline{h}_1|^2 E_1} \quad \text{and} \quad \Delta = |\underline{h}_1|^2 |\underline{h}_2|^2 - |\underline{h}_{12}|^2$$

$$\underline{\underline{B}}^H \begin{bmatrix} \underline{h}_1 & \underline{h}_2 \end{bmatrix} = \left( \begin{array}{c|c} |\underline{h}_1| & \frac{h_{12}}{|\underline{h}_1|} \\ \hline h_{12} & \sqrt{\frac{\Delta E_1 + h_2^2}{1 + h_1^2 E_1}} \end{array} \right)$$





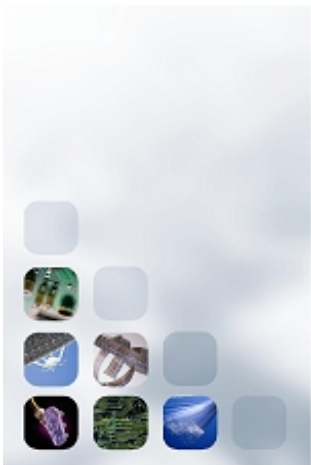
Decoding user 1 without interference of user 2 implies to decode first user 2 and then subtract the interference to user 1. Since user 2 have to be decoded with interference we will use a MSE receiver to remove interference from user 1 as much as possible.

$$MSE = E_2 \cdot \underline{h}_2^H \cdot \left( I - \frac{\underline{h}_1 \cdot \underline{h}_1^H \cdot E_1}{1 + |\underline{h}_1|^2 \cdot E_1} \right) \cdot \underline{h}_2$$

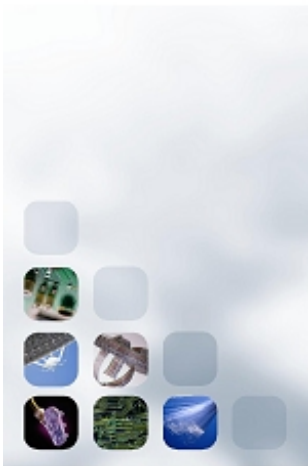
$$r2\_MSE = \log \left( 1 + \frac{|\underline{h}_2|^2 \cdot E_2 + E_1 E_2 \cdot \Delta}{1 + |\underline{h}_1|^2 E_1} \right)$$

This rate corresponds to the capacity achieving rate of the previous slide

$$r2\_MSE = r2B$$







r2

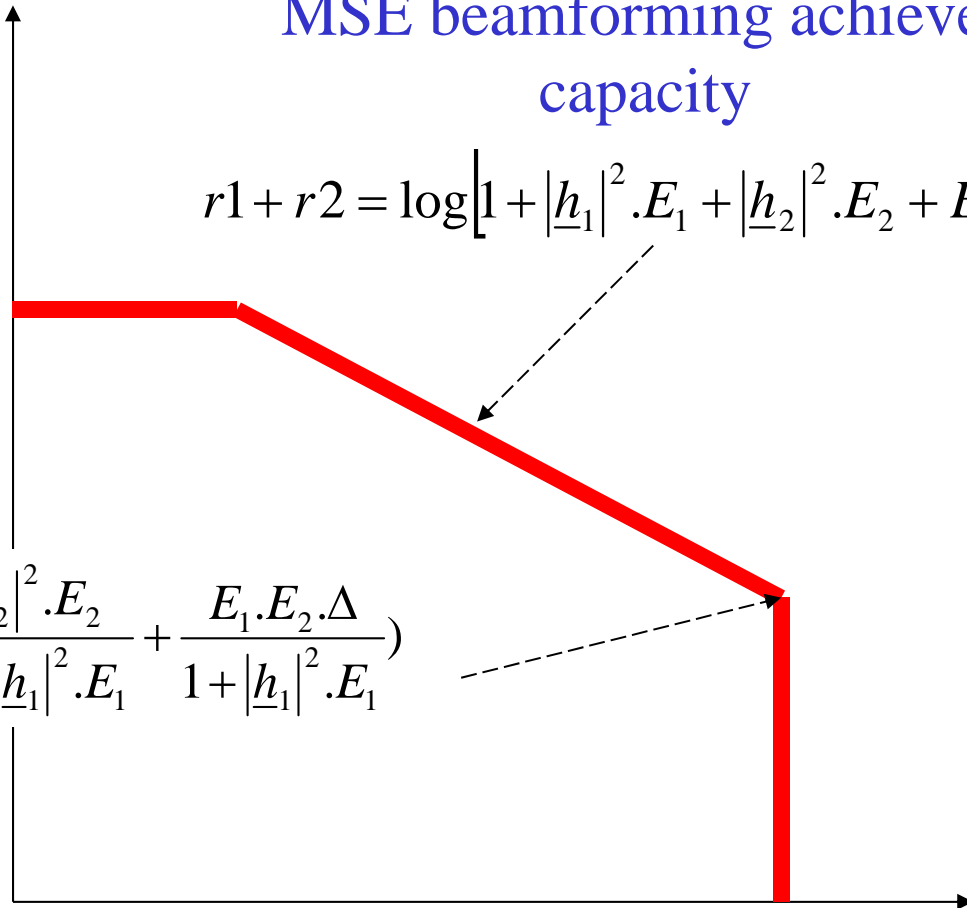
MSE beamforming achieves capacity

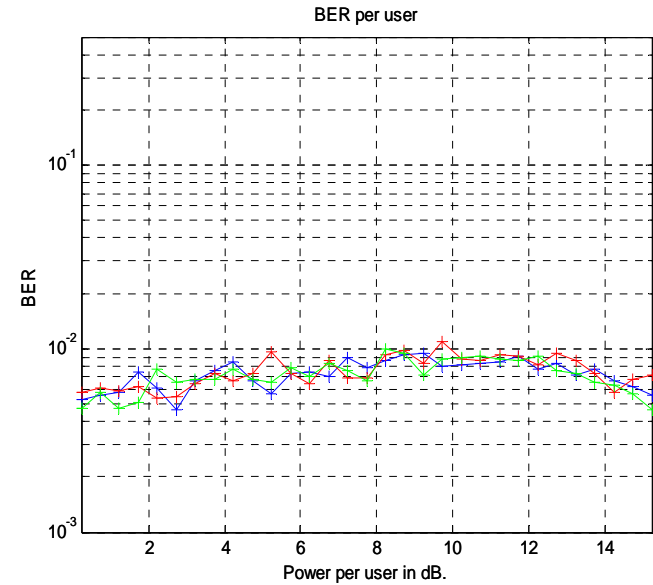
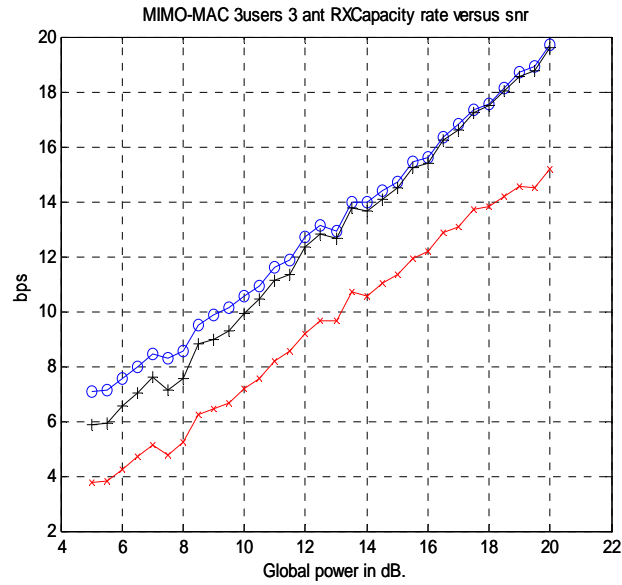
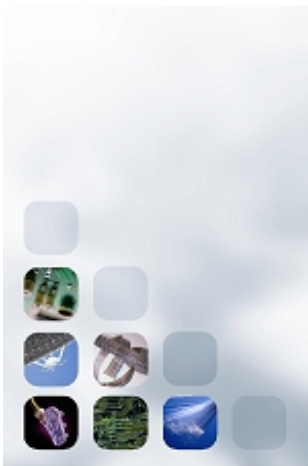
$$r1 + r2 = \log \left[ 1 + |h_1|^2 \cdot E_1 + |h_2|^2 \cdot E_2 + E_1 \cdot E_2 \cdot \Delta \right]$$

$$r2B = \log \left( 1 + \frac{|h_2|^2 \cdot E_2}{1 + |h_1|^2 \cdot E_1} + \frac{E_1 \cdot E_2 \cdot \Delta}{1 + |h_1|^2 \cdot E_1} \right)$$

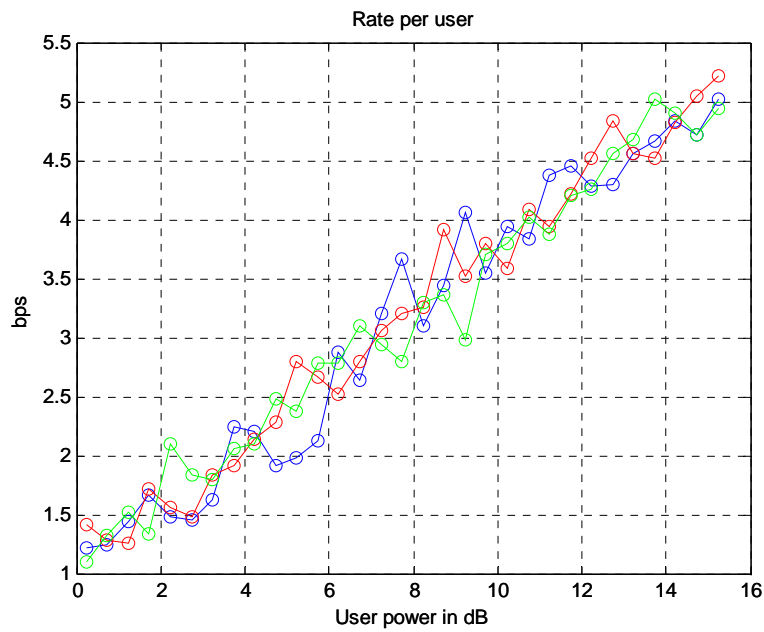
$$\log(1 + |h_1|^2 \cdot E_1)$$

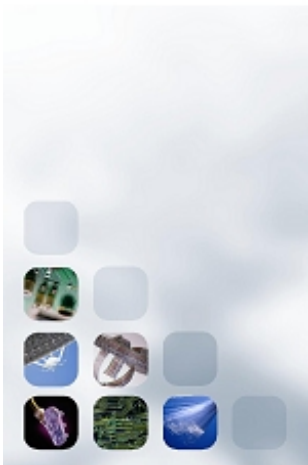
r1



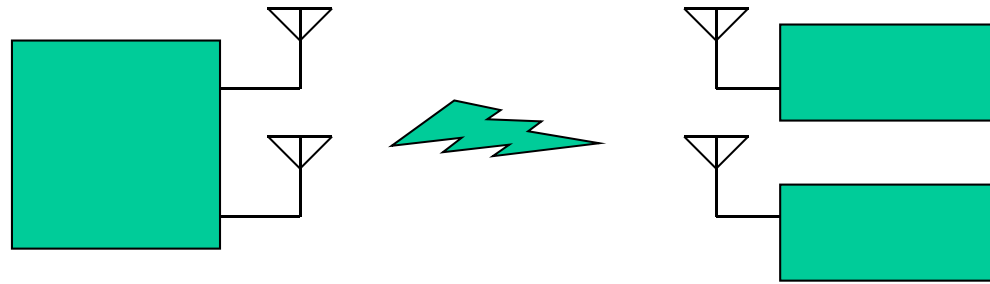


BER=10<sup>-2</sup>





## MIMO Broadcast (MIMO-BC)



The Rx cannot diagonalize the channel

The capacity region is set when forcing a global power for the Tx.

$$r_1 = \log\left(1 + |h_{-1}|^2 E_1\right) \Rightarrow \frac{2^{r_1} - 1}{|h_{-1}|^2} = E_1$$

$$r_2 = \log\left(1 + |h_{-2}|^2 E_2\right) \Rightarrow \frac{2^{r_2} - 1}{|h_{-2}|^2} = E_2$$

$$E_T = \frac{2^{r_1} - 1}{|h_{-1}|^2} + \frac{2^{r_2} - 1}{|h_{-2}|^2}$$



## BC Channel Capacity

Maximize sum-rate with the constrain of maximum Tx power

$$r_1 + r_2 = \log \left( 1 + |h_1|^2 E_1 + |h_2|^2 E_2 + \Delta E_1 E_2 \right)$$

From the PTP formula

$$s.t. \quad E_T = E_1 + E_2$$

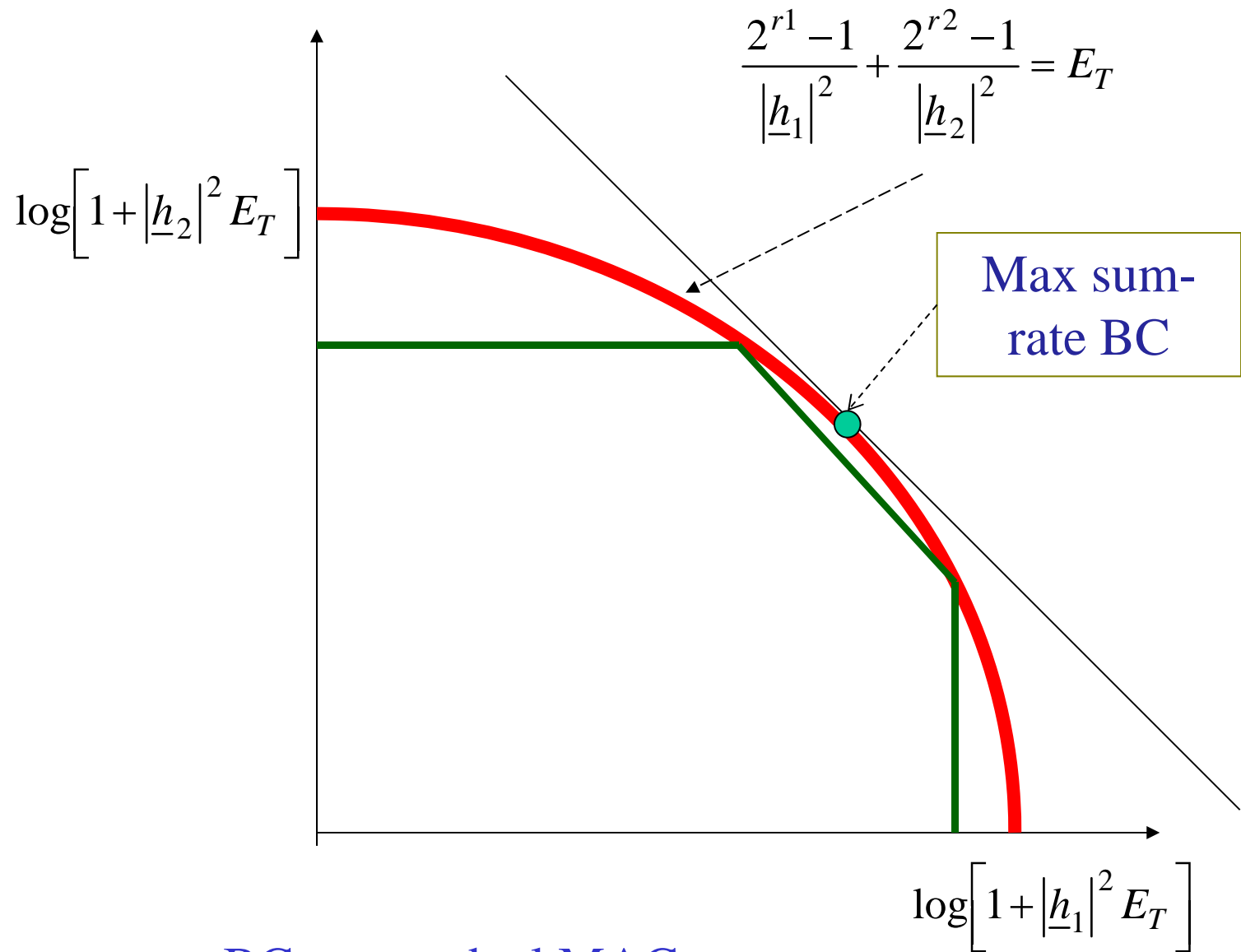
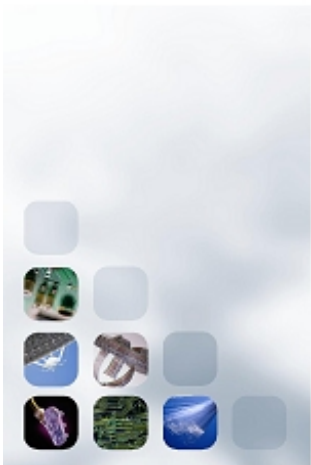
The optimum power allocation is.....

$$E_1 = \frac{E_T}{2} + \frac{|h_1|^2 - |h_2|^2}{2\Delta}$$

$$E_2 = \frac{E_T}{2} - \frac{|h_1|^2 - |h_2|^2}{2\Delta}$$

And the sum-rate maximized is

$$r_1 + r_2 \Big|_{\max} = 2 \log 2 \left( \Delta^{0.5} \frac{E_T}{2} + \frac{|h_1|^2 + |h_2|^2}{2\Delta^{0.5}} \right) - \frac{|h_{12}|^2}{\Delta}$$



BC versus dual MAC

## ZF Beamforming for MIMO\_BC

Tx diagonalizes the channel but it has to use ZF since no cooperation is available at the receivers.

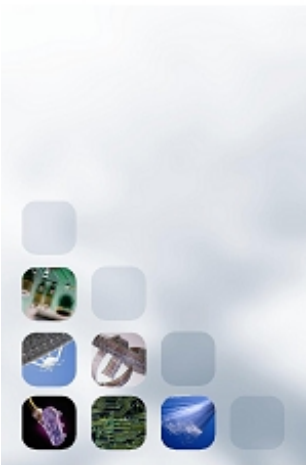
$$\underline{\underline{B}} = \left( \left[ \underline{\underline{I}} - \frac{\underline{h}_2 \underline{h}_2^H}{|\underline{h}_2|^2} \right] \frac{\underline{h}_1}{(1-\phi)^{1/2} |\underline{h}_1|} \quad \left[ \underline{\underline{I}} - \frac{\underline{h}_1 \underline{h}_1^H}{|\underline{h}_1|^2} \right] \frac{\underline{h}_2}{(1-\phi)^{1/2} |\underline{h}_2|} \right)$$

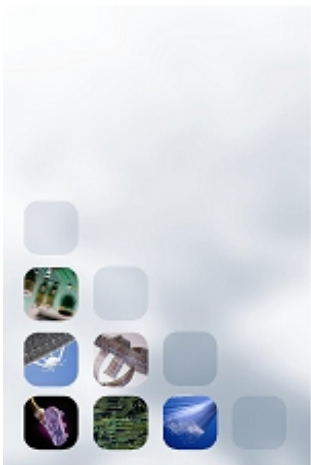
The rate achieved by every user is:

$$r_1 = \log \left[ 1 + (1-\phi) |\underline{h}_1|^2 E_1 \right]$$

$$r_2 = \log \left[ 1 + (1-\phi) |\underline{h}_2|^2 E_2 \right]$$

$$\frac{2^{r_1} - 1}{|\underline{h}_1|^2} + \frac{2^{r_2} - 1}{|\underline{h}_2|^2} = (1-\phi) E_T$$





And the ZF max. sum-  
rate is:

$$r_1 + r_2 = \log \left[ 1 - \left( \frac{|h_{12}|^2}{|h_1|^2 |h_2|^2} \right) \left( |h_1|^2 E_1 + |h_2|^2 E_2 + \Delta E_1 E_2 \right) \right]$$

Loss due to the use of  
Zero-Forcing  
Beamforming



## Using lower triangular zero forcing

$$\underline{\underline{B}} = \left( \underline{h}_1 \quad \left[ \underline{I} - \frac{\underline{h}_1 \underline{h}_1^H}{|\underline{h}_1|^2} \right] \frac{\underline{h}_2}{(1-\phi)^{1/2} \cdot |\underline{h}_2|} \right)$$

Detect stream 1 precancel interference of stream 1 on stream 2

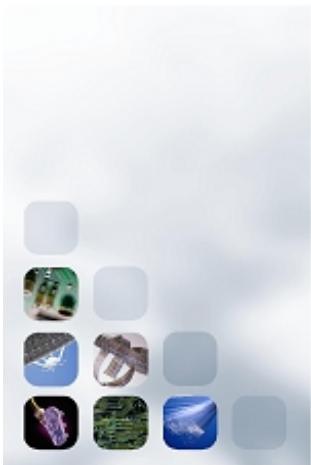


$$r1 = \log \left[ 1 + |\underline{h}_1|^2 E_1 \right]$$

$$r2 = \log \left[ 1 + (1-\phi) |\underline{h}_2|^2 E_2 \right]$$

Later on it will be explained how the interference term is removed

$$\frac{2^{r1}}{|\underline{h}_1|^2} + \frac{2^{r2}}{(1-\phi) |\underline{h}_2|^2} = E_T + \left( \frac{1}{|\underline{h}_1|^2} + \frac{1}{(1-\phi) \cdot |\underline{h}_2|^2} \right)$$

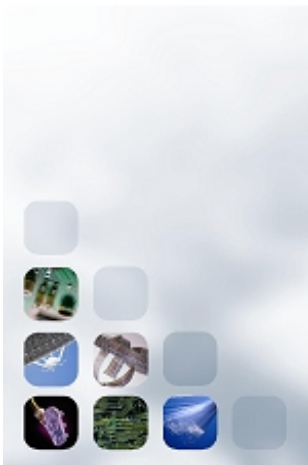




The sum-rate is:

$$r_1 + r_2 = \log \left[ 1 + |h_1|^2 E_1 + (1 - \phi) |h_2|^2 E_2 + \Delta E_1 E_2 \right]$$

Loss due for the “semi-ZF” or QR beamforming and “writing in a dirty space”



## DPC A Signal Processing Formulation

From the likelihood, we can define the following error

$$\underline{R}_0^{-0.5} \left( \underline{X}_{Rn} - \underline{H} \underline{B} \underline{s}_n \right) \Rightarrow |\underline{\varepsilon}_n|^2 = \left| \underline{R}_0^{-0.5} \underline{X}_{Rn} - \underline{R}_0^{-0.5} \underline{H} \underline{B} \underline{s}_n \right|^2$$

Now, using the QR decomposition for the global channel

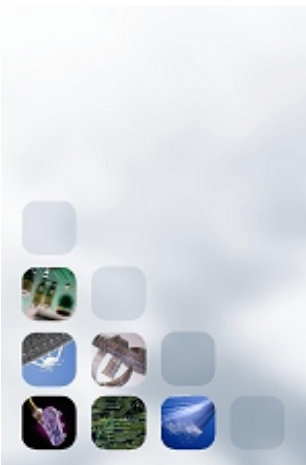
$$\underline{R}_0^{-0.5} \underline{H} = \underline{R}^H \underline{Q}^H$$

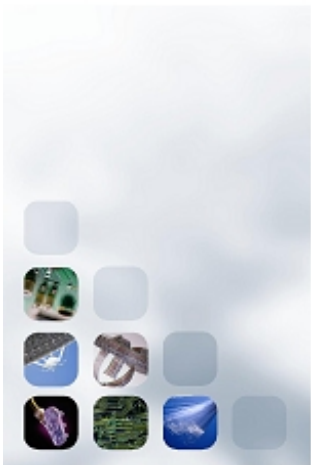
Then, using the beamforming matrix B equal to Q, we have:

$$\underline{B} = \underline{Q} \quad \underline{\varepsilon}_n = \underline{Y}_n - \underline{R}^H \underline{s}_n \quad \text{where} \quad \underline{Y}_n = \underline{R}_0^{-0.5} \underline{X}_{Rn}$$

Now.....

with  $\underline{R}^H = \text{diag} \left( \underline{R}^H \right) \underline{R}_a$  being  $\text{diag} \left( \underline{R}_a \right)$  is the identity matrix





Let us define the input streams vector as  $\underline{s}_n = \underline{R}_a^{-1} \underline{v}_n$

We have

$$\underline{s}_n = \underline{R}_a^{-1} \underline{v}_n$$

$$\underline{R}_a \underline{s}_n = \underline{v}_n$$

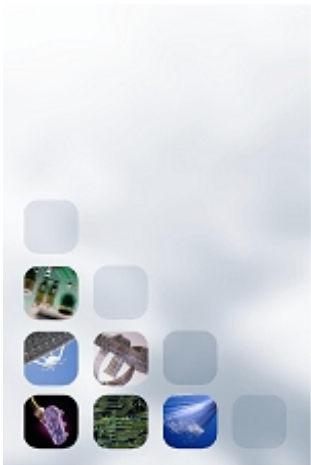
$$\text{diag}(\underline{R}_H) = \underline{D}$$

$$\underline{\varepsilon}_n = \underline{Y}_n - \underline{D} \underline{R}_a \underline{s}_n = \underline{Y}_n - \underline{D} \underline{v}_n$$

This is the AGC of  
each receiver

!!! The receiver can be  
decentralized, so each receiver  
decodes its message!!!

Thus if the input is vector  $\underline{v}_n$  instead of  $\underline{s}_n$  we solve the  
problem for the BC scenario

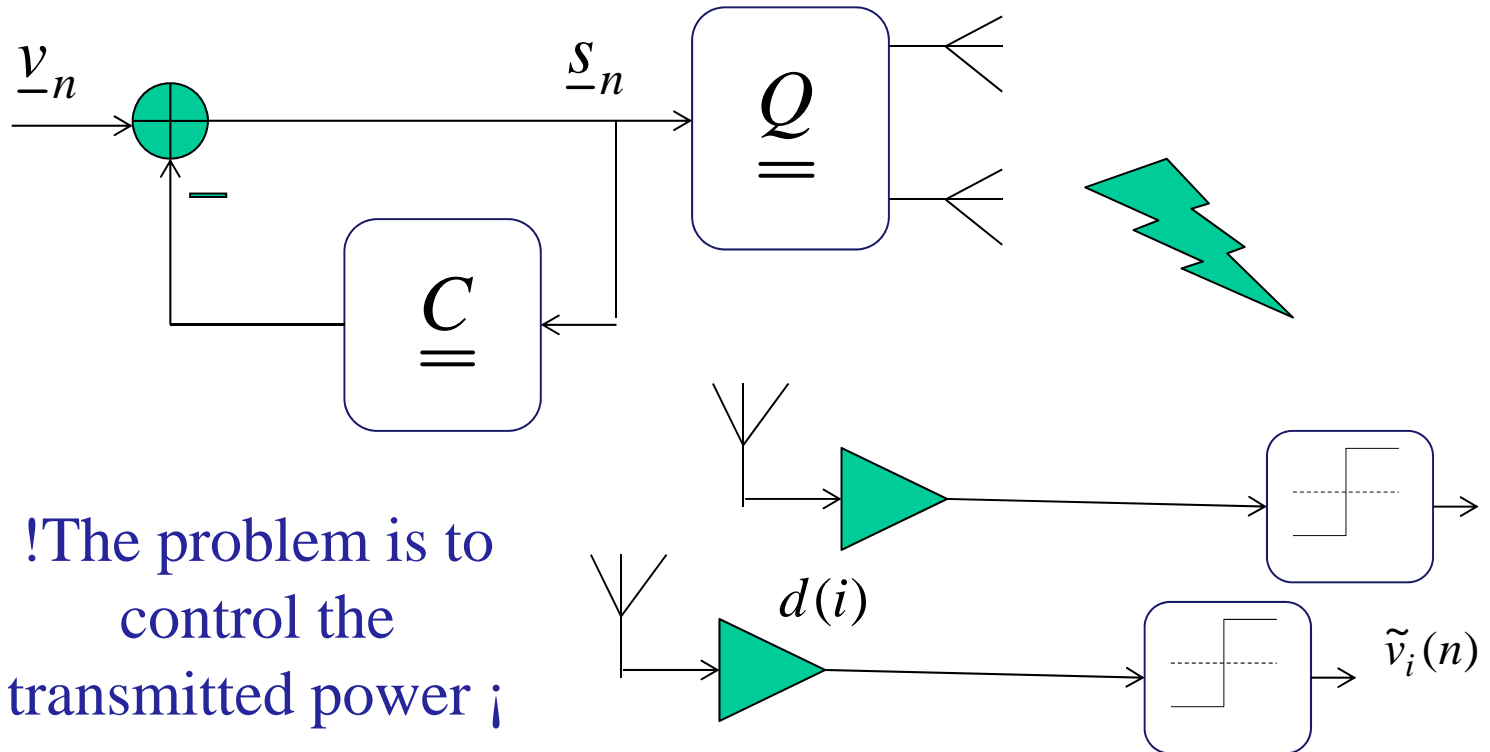


Since.....  $\underline{s}_n = \underline{R}_a^{-1} \underline{v}_n$

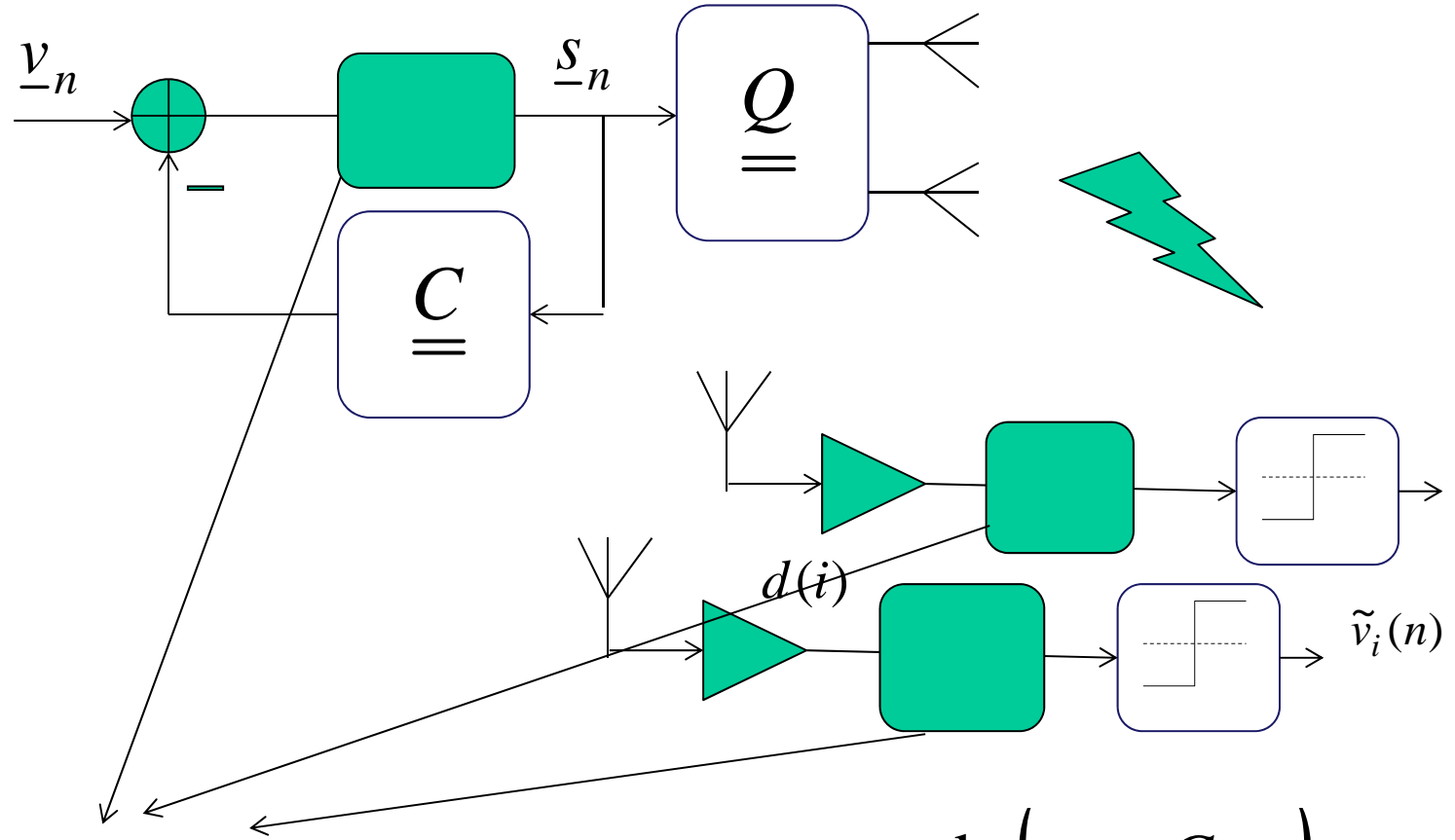
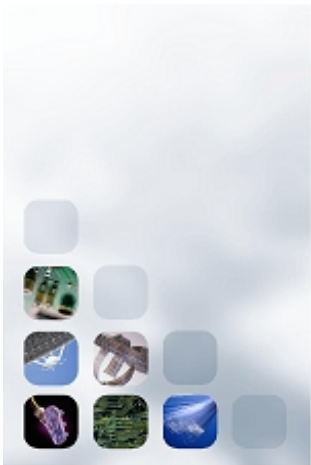
$$\underline{R}_a = \underline{I} + \underline{C}$$

This matrix is strict lower triangular

then  $\underline{s}_n + \underline{C}\underline{s}_n = \underline{v}_n \Rightarrow \underline{s}_n = \underline{v}_n - \underline{C}\underline{s}_n$



!The problem is to control the transmitted power ;

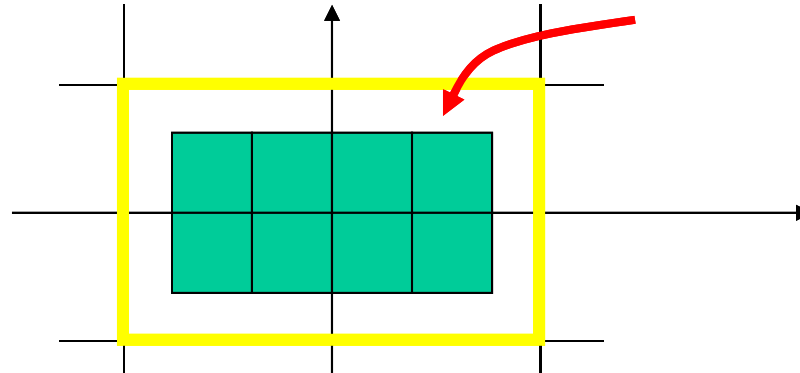


Modulus  
operation

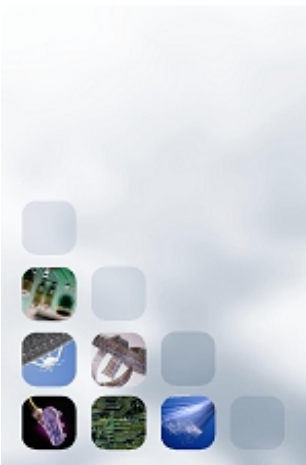
$$\underline{s}_n = \text{mod}_L(\underline{v}_n - \underline{C}\underline{s}_n)$$

$$E_T \approx \frac{L^2}{3}$$

The modulus operation have to be introduced independently  
For in-phase and quadrature components of the constellation



Thresholds for  
modulus operation  
on a 8QAM  
constellation



The uniform distribution within the limits of the basic modulus operation (thresholds set equal to the maximum plus half the symbol separation distance) provides a power equal to the square of the maximum divided by 3.

Modulation	Modulus at:			
	Unit Power	In-phase	Quadrat.	Excess Power
BPSK	2	0	0	1.24 dB.
QPSK	$2/\sqrt{2}$	$2/\sqrt{2}$	0	1.24 dB.
8-QAM	$4/\sqrt{6}$	$2/\sqrt{2}$	0	0.45 dB.
16-QAM	$4/\sqrt{10}$	$4/\sqrt{10}$	0	0.28 dB.
32-QAM	$8/\sqrt{26}$	$4/\sqrt{10}$	0	0.11 dB.
64-QAM	$8/\sqrt{42}$	$8/\sqrt{42}$	0	0.06 dB.

The users have to be re-labeled such that the diagonal terms of the R matrix are in strict decreasing order.

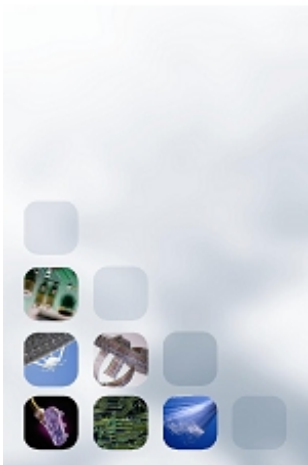
To solve the labeling problem we need the generalized qr decomposition, where matrix T is a permutation that provides the diagonals of R in decreasing order. This is in accordance with the scheduling, whenever power is included within the QR.

$$\underline{\underline{R}}_0^{-0.5} \cdot \underline{\underline{H}} = \underline{\underline{Q}} \cdot \underline{\underline{R}} \cdot \underline{\underline{T}}^H$$

The new error is:

$$\underline{\underline{R}}_0^{-0.5} \cdot \underline{\underline{X}}_{Rn} - \underline{\underline{R}}_0^{-0.5} \cdot \underline{\underline{H}} \cdot \underline{\underline{B}}_{S_n} = \underline{\underline{R}}_0^{-0.5} \cdot \underline{\underline{X}}_{Rn} - \text{diag}(\text{diag}(\underline{\underline{R}})) \cdot \underline{\underline{R}}_a \cdot \underline{\underline{T}}^H \underline{\underline{S}}_n$$

and



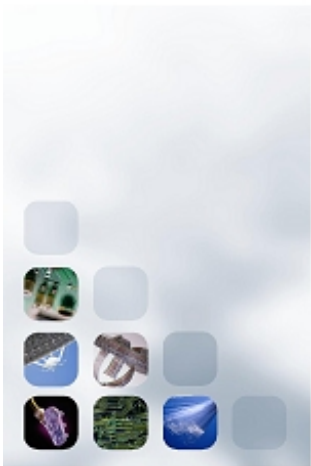
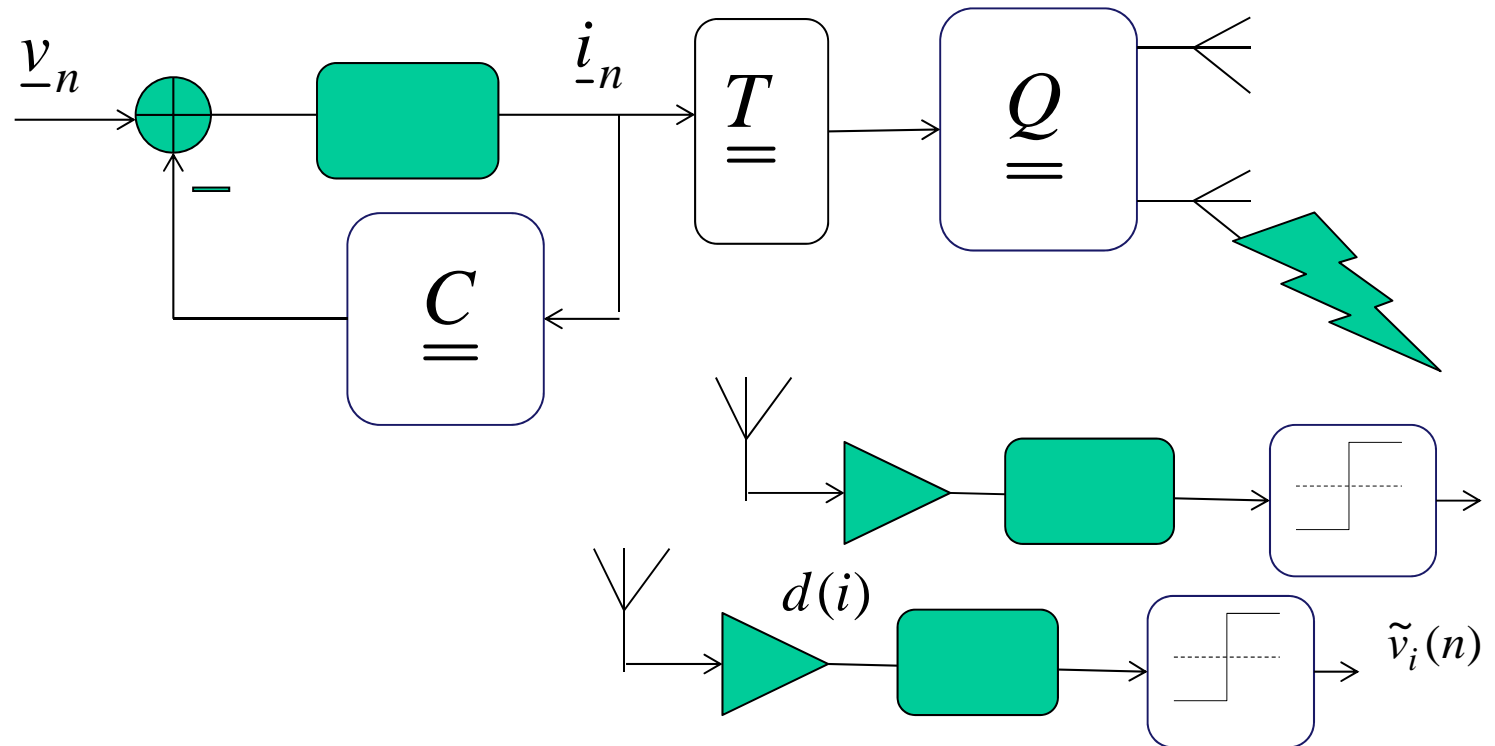


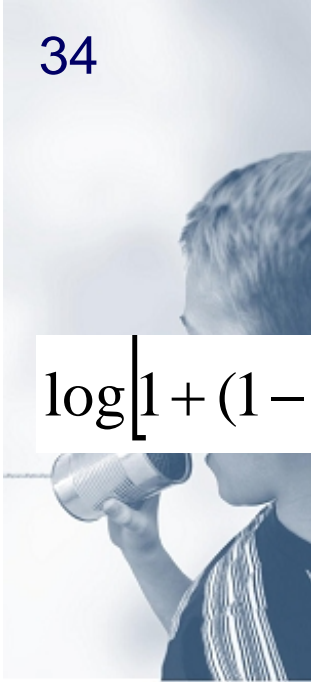


$$\underline{R}_{=0}^{-0.5} \cdot \underline{X}_{Rn} - \underline{Q} \cdot \text{diag}(\text{diag}(\underline{R})) \cdot \underline{R}_{=a} \cdot \underline{T}^H \underline{T} \underline{i}_{=n}$$

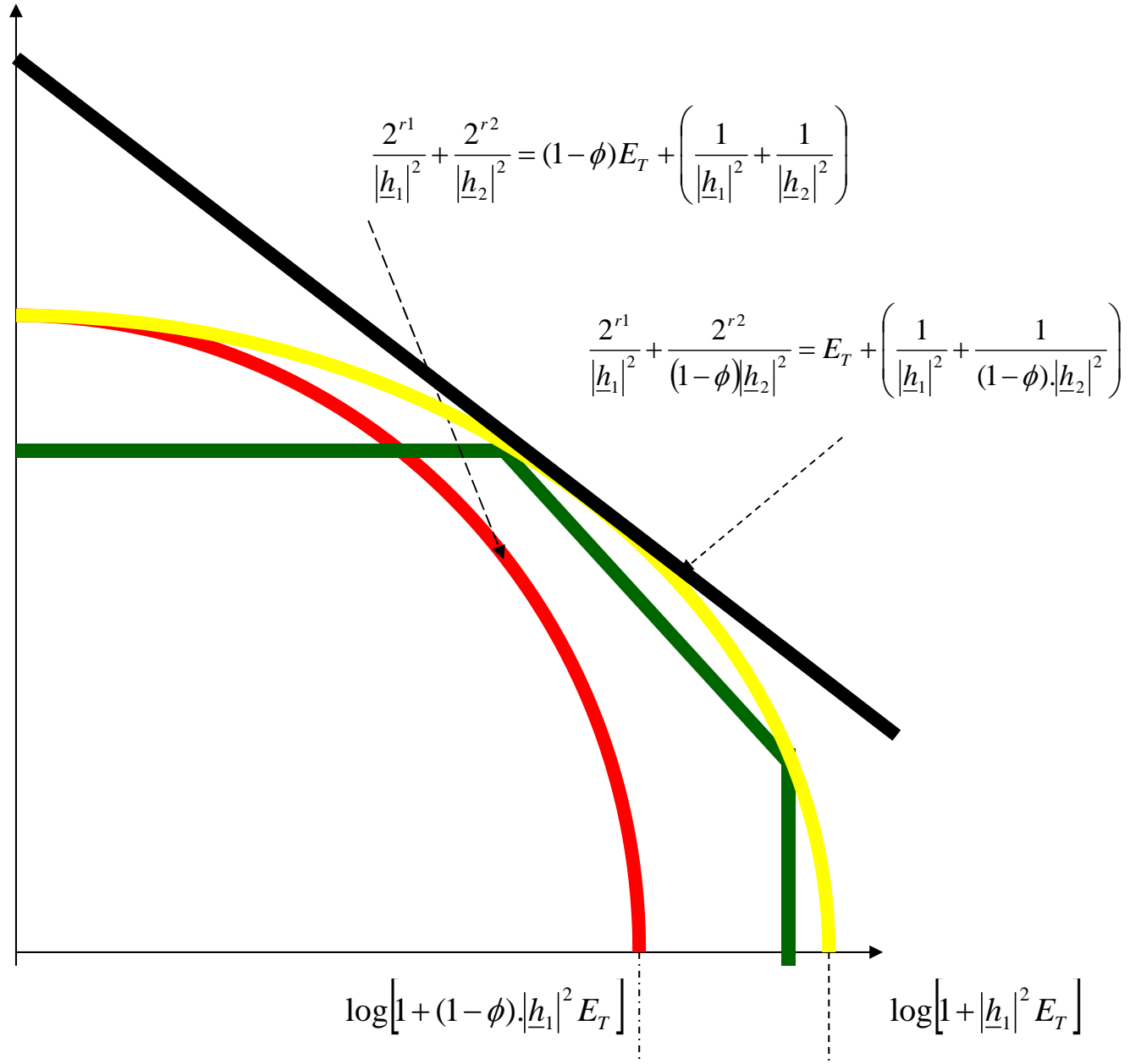
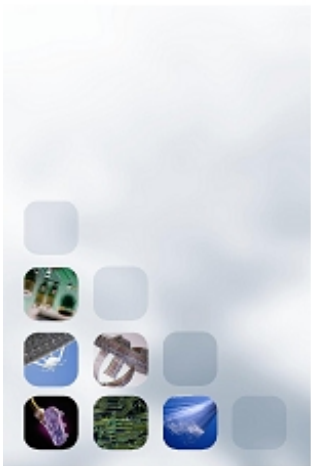
$$\underline{s}_{=n} = \underline{T} \underline{i}_{=n} \Rightarrow \underline{i}_{=n} = \underline{T}^H \underline{s}_{=n}$$

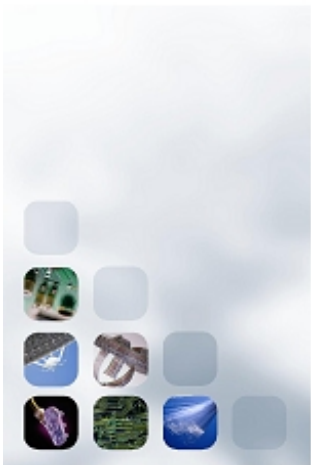
Thus the final scheme is:





$$\log \left[ 1 + (1 - \phi) |h_2|^2 E_T \right]$$





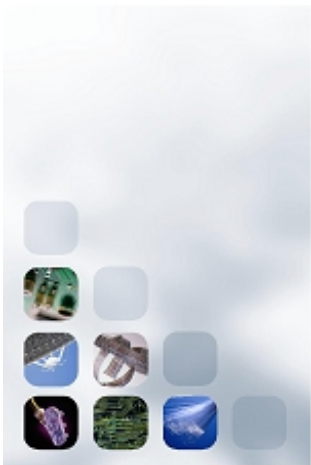
## MSE Beamforming

$$\underline{\underline{B}} = \left( \frac{\underline{h}_1}{|\underline{h}_1|} \left[ I - \frac{E_1 \underline{h}_1 \underline{h}_1^H}{1 + E_1 |\underline{h}_1|^2} \right] \frac{\underline{h}_2}{(1 - \phi)^{1/2} |\underline{h}_2|} \right)$$

$$\underline{b}_2^H \underline{b}_2 = \frac{(h_2^2 + \Delta E_1)(1 + h_1^2 E_1) - h_{12}^2 E_1}{(1 + h_1^2 E_1)^2}$$

$$\underline{\underline{HB}} = \begin{pmatrix} \underline{h}_1^H \\ \underline{h}_2^H \end{pmatrix} \underline{\underline{B}} = \begin{pmatrix} h_{1b1} & h_{1b2} \\ h_{2b1} & h_{2b2} \end{pmatrix}$$

Removed by the  
TH precoder



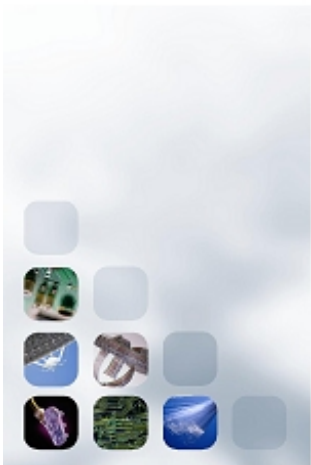
$$SNR1 = \frac{h_1^2 E_1^{BC}}{1 + (h_{12})^2 E_2^{BC}} = \frac{h_1^2 E_1^{BC}}{1 + \Phi^2 E_2^{BC}}$$

$$h_{12} = \frac{(h_{12} / 1 + h_1^2 E_1)}{(\dots\dots)} = \frac{h_{12}}{\sqrt{(\Delta E_1 + h_2^2) + \Delta E_1 (1 + h_1^2 E_1)}} = \Phi$$

$$1 - \Phi^2 E_1 = \frac{(\Delta E_1 + h_2^2)(1 + h_1^2 E_1)}{(\Delta E_1 + h_2^2) + \Delta E_1 (1 + h_1^2 E_1)}$$

$$SNR1 = \frac{h_1^2 E_1^{BC}}{1 + (h_{12})^2 E_2^{BC}} = \frac{h_1^2 E_1^{BC}}{1 + \Phi^2 E_2^{BC}} = h_1^2 E_1$$

$$E_1^{BC} = E_1 (1 + \Phi^2 E_2^{BC})$$



$$SNR_2 = (h_2 b_2)^2 E_2^{BC} = \frac{\Delta E_1 + h_2^2}{1 + h_1^2 E_1} E_2$$

$$(h_2 b_2)^2 = \frac{\left( \frac{\Delta E_1 + h_2^2}{1 + h_1^2 E_1} \right)^2}{\frac{(1 + h_1^2 E_1)(\Delta E_1 + h_2^2) - h_{12}^2 E_1}{(1 + h_1^2 E_1)^2}}$$

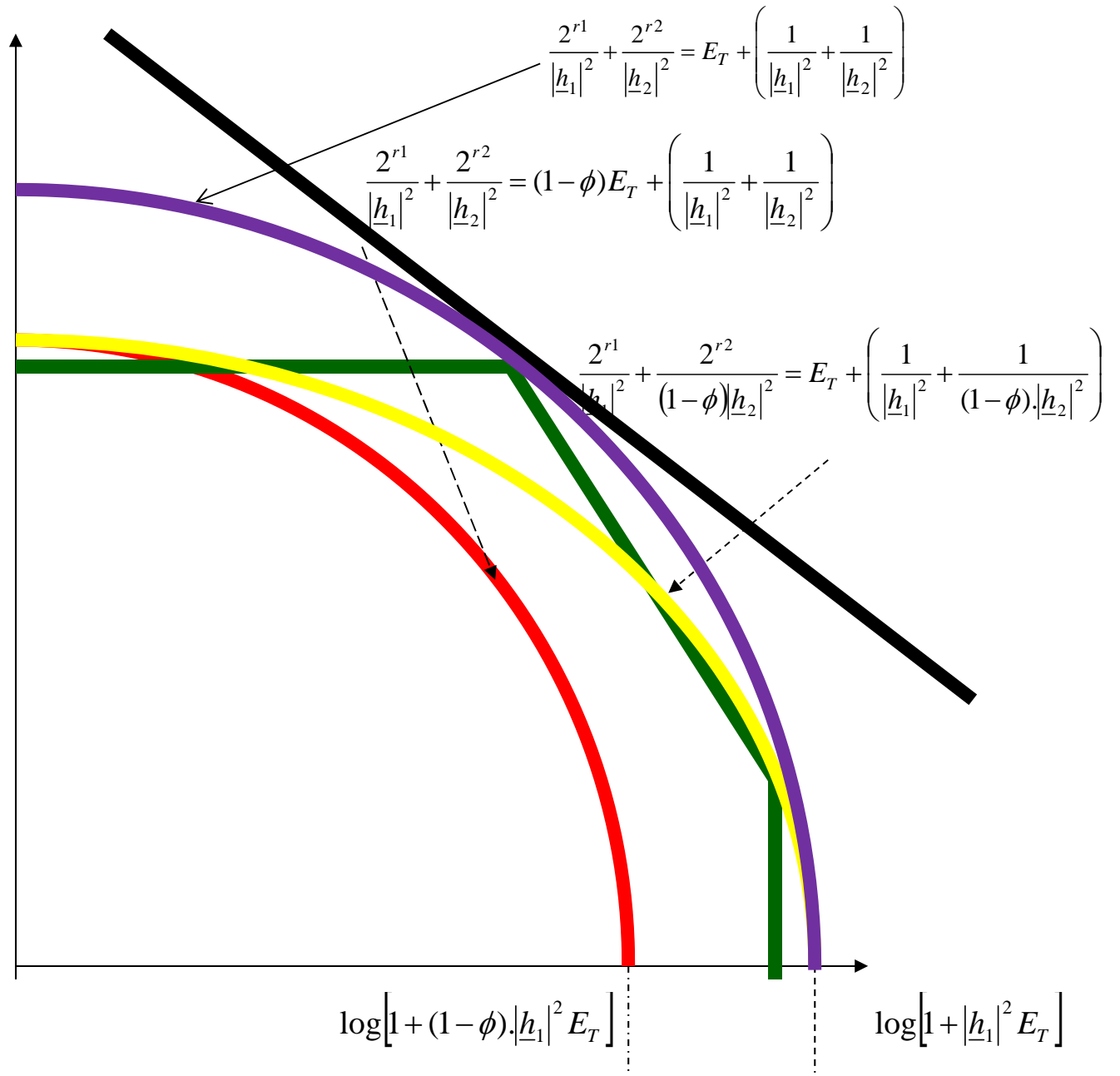
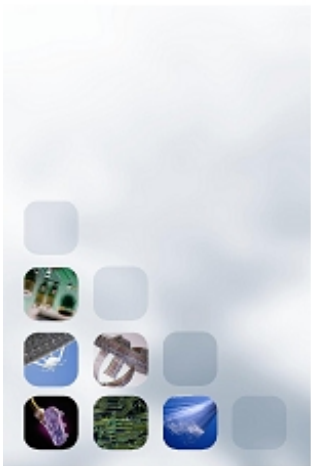
$$E_2^{BC} = \left( 1 - \frac{h_{12}^2 E_1}{(1 + h_1^2 E_1)(\Delta E_1 + h_2^2)} \right) E_2 \Rightarrow E_2^{BC} = (1 - \Gamma^2 E_1) E_2$$

$$E_1^{BC} = E_1 (1 + \Phi^2 E_2^{BC}) \quad \text{with} \quad \Phi^2 = \frac{\Gamma^2}{1 - \Gamma^2 E_1} \Rightarrow E_1^{BC} = (1 + \Gamma^2 E_2) E_1$$

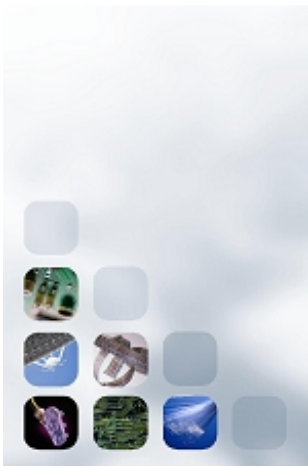
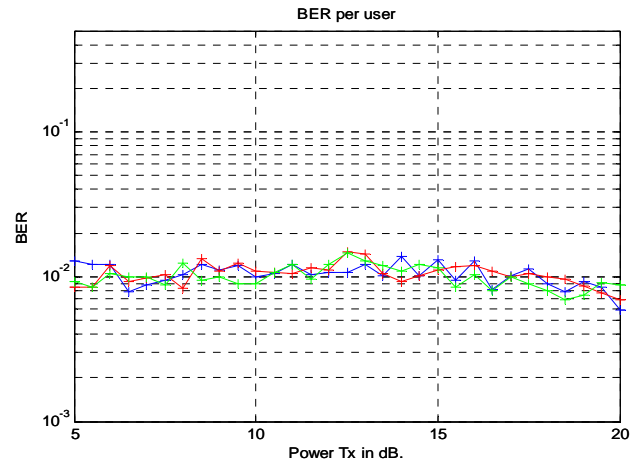
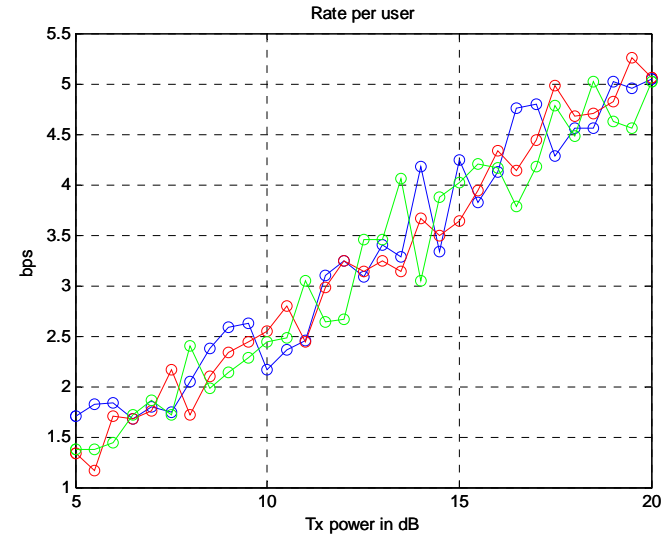
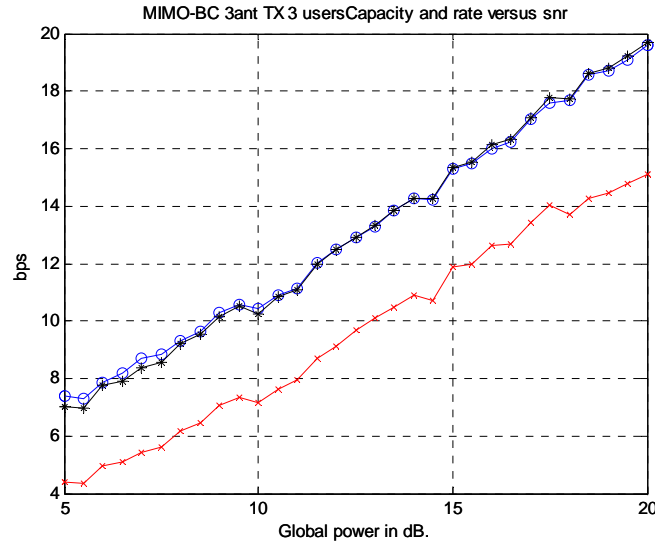


$$\log\left[1 + |\underline{h}_2|^2 E_T\right]$$


$$\log\left[1 + (1 - \phi)|\underline{h}_2|^2 E_T\right]$$



# MIMO-BC: Broadcast 3 users from a BS with 3 antennas, BER target equal to $10^{-2}$ . Transposition and modulus operation at Tx and Rx, 50 channel realizations.







innovating communications

# The Centre Tecnològic de Telecomunicacions de Catalunya

*A gateway to advanced communication technologies*



**Centre Tecnològic de Telecomunicacions de Catalunya**  
Parc Mediterrani de la Tecnologia (PMT)  
Av. Canal Olímpic S/N 08860 – Castelldefels Barcelona (Spain)

Voice: +34 93 645 29 00 :: Fax: +34 93 645 29 01

E-mail: [info@cttc.es](mailto:info@cttc.es) :: Web: <http://www.cttc.es>