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MIMO1: Single symbol over flat fading CSIT/CSIR

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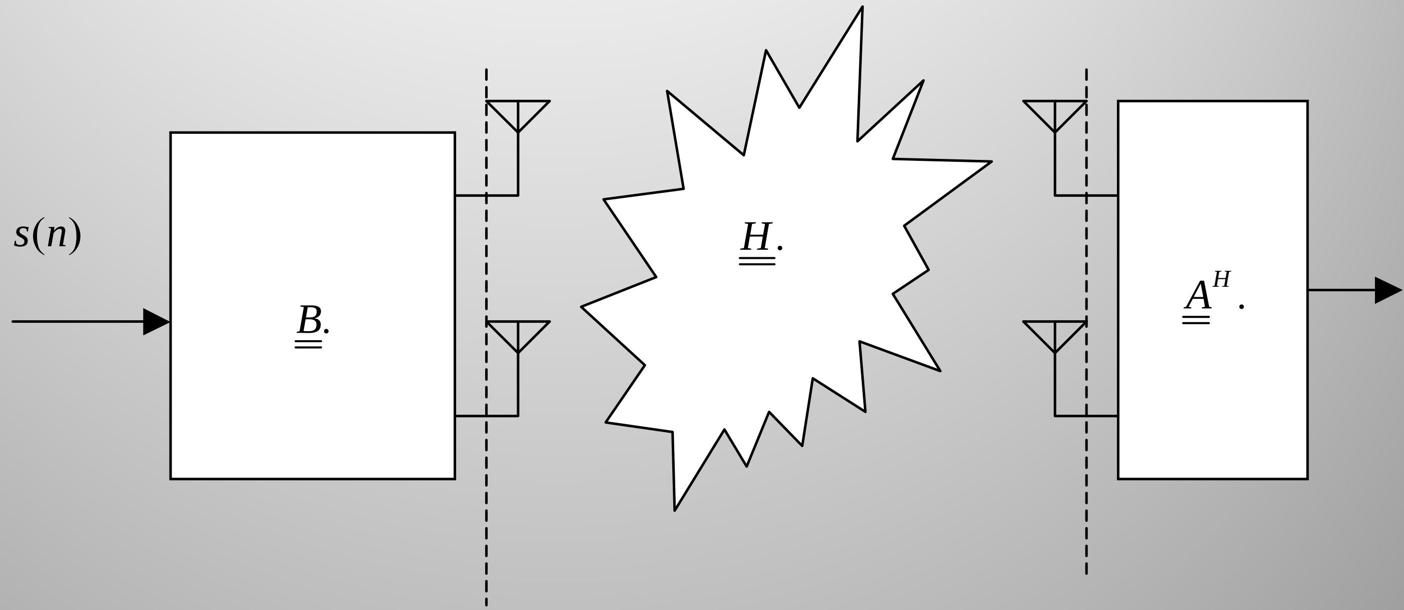
The Scenario



$$\underline{\underline{X}}_{T,n} = \underline{\underline{B}}.s(n)$$

$$\underline{\underline{X}}_{R,n} = \underline{\underline{H}}.\underline{\underline{B}}.s(n) + \underline{\underline{W}}_{n}$$

Flat Fading, CSIT and CSIR, Single Symbol

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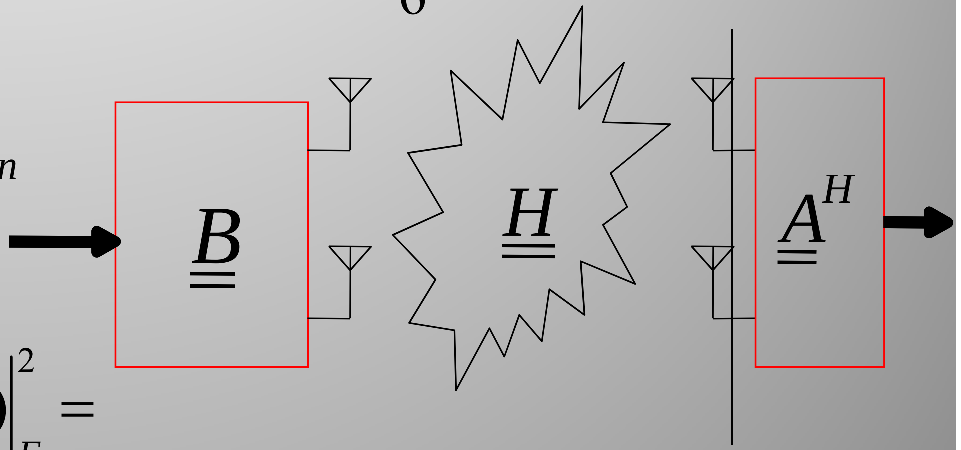
The ML Receiver

$$\underline{\underline{X}}_{T,n} = \underline{\underline{B}}.s(n)$$

$$E_T = trace\left(\underline{\underline{B}}.\underline{\underline{B}}^H\right).E\left(\left|s(n)\right|^2\right) = d^2.\frac{2^{n_s}-1}{6}.trace\left(\underline{\underline{B}}.\underline{\underline{B}}^H\right)$$

$$\underline{\underline{X}}_{R,n} = \underline{\underline{H}}.\underline{\underline{B}}.s(n) + \underline{\underline{W}}_n$$



The likelihood

$$\Lambda\left(s(n)\right) = -\left|\underline{\underline{X}}_{R,n} - \underline{\underline{H}}.\underline{\underline{B}}.s(n)\right|_F^2 =$$

$$= -Traza\left[\left(\underline{\underline{X}}_{R,n} - \underline{\underline{H}}.\underline{\underline{B}}.s(n)\right).\left(\underline{\underline{X}}_{R,n} - \underline{\underline{H}}.\underline{\underline{B}}.s(n)\right)^H\right] =$$

$$\rightarrow 2.\mathrm{Re}\left[s(n)^*.Traza\left(\underline{\underline{B}}^H.\underline{\underline{H}}^H.\underline{\underline{X}}_{R,n}\right)\right] - \left|s(n)\right|^2.Traza\left[\underline{\underline{B}}^H.\underline{\underline{H}}^H.\underline{\underline{H}}.\underline{\underline{B}}\right]$$

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ML Receiver

$$\underline{\underline{A}} = \underline{\underline{H}}.\underline{\underline{B}}$$ Optimum receiver

$$when \quad \Lambda(s(n)) > \Lambda(s(m)) \quad \forall m \neq n \quad decide \quad s(n)$$

After some manipulations and with $\tilde{s}(n) = s(n) - s(m)$

$$Traza\left[\left(\underline{\underline{B}}^H.\underline{\underline{H}}^H.\underline{\underline{H}}.\underline{\underline{B}}\right).\left|\tilde{s}(n)\right|^2\right] > 2.\mathrm{Re}\left[\tilde{s}(n).Traza\left(\underline{\underline{W}}_n^H\,\underline{\underline{H}}.\underline{\underline{B}}\right)\right]$$

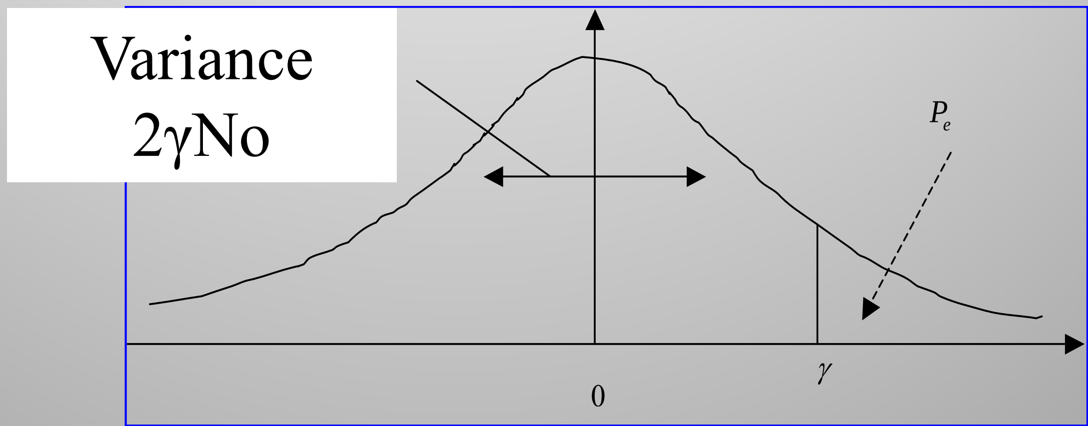
The global Tx-H matrix

define

$$\gamma = d^2.Trace\left(\underline{\underline{H}}^H.\underline{\underline{R}}_0^{-1}.\underline{\underline{H}}.\underline{\underline{B}}.\underline{\underline{B}}^H\right) = d^2.Trace\left[\underline{\underline{R}}_H.\left(\underline{\underline{B}}.\underline{\underline{B}}^H\right)\right]$$

The Probability of Error

$$Traza\left[\left(\underline{\underline{B}}^{H}.\underline{\underline{H}}^{H}.\underline{\underline{H}}.\underline{\underline{B}}\right).\left|\tilde{s}(n)\right|^{2}\right] > 2.\mathrm{Re}\left[\tilde{s}(n).Traza\left(\underline{\underline{W}}_{n}^{H}\,\underline{\underline{H}}.\underline{\underline{B}}\right)\right]$$



Error only to the nearest neighborhood

$$\Pr(s(n)\rightarrow s(m)) = P_e = Q\left(\sqrt{\frac{\gamma}{2.N_0}}\right)$$

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Tx Optimization

$$d^2 trace\left(\underline{\underline{R}}_H . \underline{\underline{B}} . \underline{\underline{B}}^H\right)\Big|_{MAX}$$

$$s.t. \quad E_s . trace\left(\underline{\underline{B}} . \underline{\underline{B}}^H\right) = E_T$$

Max. Quality with a given power budget

The channel matrix

$$\underline{\underline{R}}_H = \underline{\underline{H}}^H \underline{\underline{R}}_0^{-1} . \underline{\underline{H}}$$

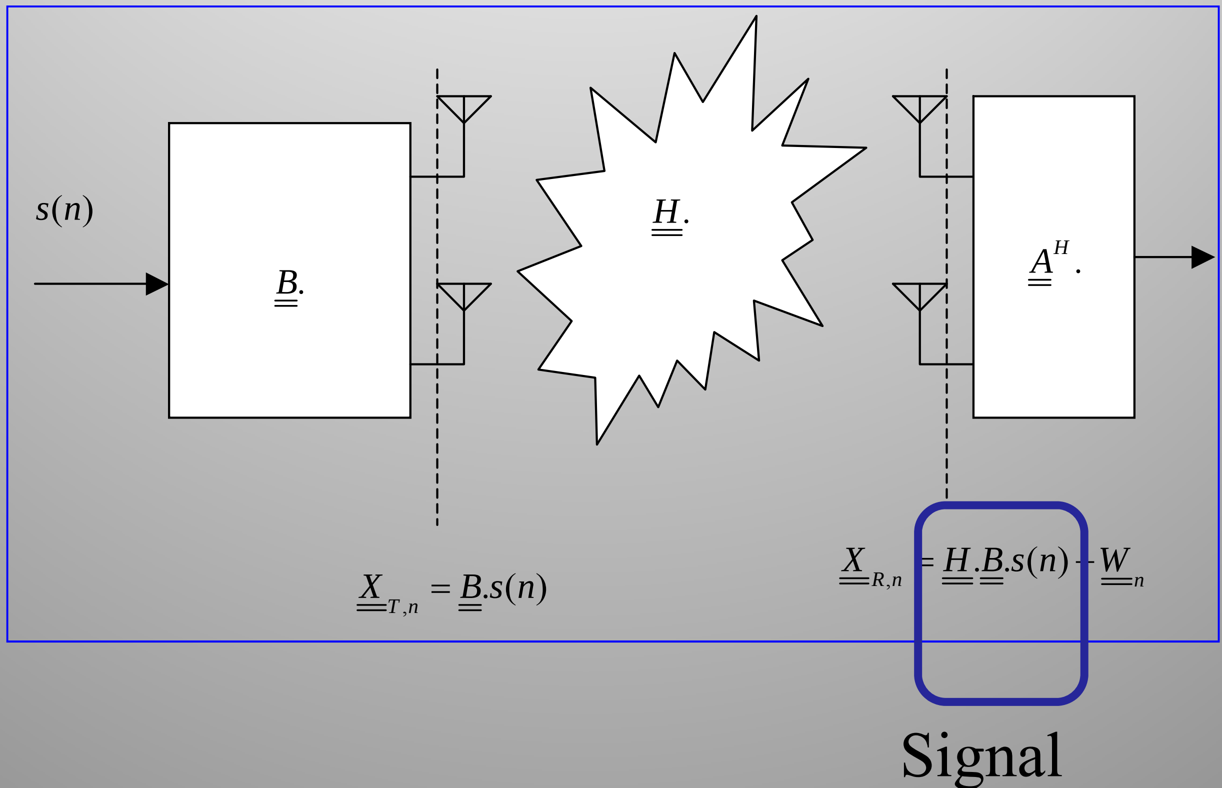
The Tx matrix and constraint

$$\underline{\underline{Q}} = (\underline{\underline{B}} . \underline{\underline{B}}^H) . E_s = (\underline{\underline{B}} . \underline{\underline{B}}^H) . d^2 . \frac{2^{n_b} - 1}{6}$$

$$Trace\left(\underline{\underline{Q}}\right) = d^2 . \frac{2^{n_b} - 1}{6} . Trace(\underline{\underline{B}} . \underline{\underline{B}}^H) \leq E_T$$

Maximum SNR Design (ML)

An alternative design (easy to perform) is to compute the SNR at the receiver output and then maximise it for the receiver and the transmitter.



At the receiver the estimated symbol will be:

$$\hat{s}(n) = trace\left(\underline{\underline{A}}^H \underline{\underline{H}}\,\underline{\underline{B}}\right)s(n) + trace\left(\underline{\underline{A}}^H \underline{\underline{W}}_n\right)$$

In consequence, the SNR will be:

$$SNR = \frac{tr^2\left(\underline{\underline{A}}^H \underline{\underline{H}}\,\underline{\underline{B}}\right)E_s}{tr\left(\underline{\underline{A}}^H \underline{\underline{R}}_0 \underline{\underline{A}}\right)}$$

Since the trace is a linear operator
we use the SW un-equality as:

$$tr\left(\underline{\underline{u}}^H \underline{\underline{u}}\right)tr\left(\underline{\underline{v}}^H \underline{\underline{v}}\right) \leq tr^2\left(\underline{\underline{u}}^H \underline{\underline{v}}\right)$$

with
$$\underline{\underline{u}} = \underline{\underline{R}}_0^{1/2} \underline{\underline{A}}$$
$$\underline{\underline{v}} = \underline{\underline{R}}_0^{-1/2} \underline{\underline{H}}\,\underline{\underline{B}}$$

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$$SNR = \frac{tr^2\left(\underline{\underline{A}}^H \underline{\underline{H}}\,\underline{\underline{B}}\right)E_s}{tr\left(\underline{\underline{A}}^H \underline{\underline{R}}_0 \underline{\underline{A}}\right)} = \frac{tr^2\left(\underline{\underline{u}}^H \underline{\underline{v}}\right)E_s}{tr\left(\underline{\underline{u}}^H \underline{\underline{u}}\right)} \leq E_s tr\left(\underline{\underline{v}}^H \underline{\underline{v}}\right)$$

Thus the maximum SNR is:

$$SNR = E_s tr\left(\underline{\underline{B}}^H \left[\underline{\underline{H}}^H \underline{\underline{R}}_0^{-1} \underline{\underline{H}}\right]\underline{\underline{B}}\right) = E_s tr\left(\underline{\underline{B}}^H \underline{\underline{R}}_C \underline{\underline{B}}\right)$$

This maximum is achieved when both matrixes are proportional, then……

The channel matrix (gain/noise)

$$\underline{\underline{R}}_0^{1/2} \underline{\underline{A}} \propto \underline{\underline{R}}_0^{-1/2} \underline{\underline{H}}\,\underline{\underline{B}} \quad \Rightarrow \quad \boxed{\underline{\underline{A}} = \underline{\underline{R}}_0^{-1} \underline{\underline{H}}\,\underline{\underline{B}}}$$

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Next, the SNR have to be maximized with respect the transmitter matrix B

$$SNR = E_s tr\left(\underline{\underline{B}}^H \underline{\underline{R}}_C \underline{\underline{B}}\right)$$

Subject to the Tx power constraint

$$s.t. \quad E_s .trace\left(\underline{\underline{B}}.\underline{\underline{B}}^H\right) = E_T$$

To solve this problem we will use the following property:

$$trace\left(\underline{\underline{R}}_C \underline{\underline{B}}.\underline{\underline{B}}^H\right) \leq \lambda_{\max}\left(\underline{\underline{R}}_C\right) tr\left(\underline{\underline{B}}\underline{\underline{B}}^H\right) = \lambda_{\max}\left(\underline{\underline{R}}_C\right)\frac{E_T}{E_s}$$

In summary, the maximum SNR is….

$$SNR = E_T \lambda_{\max}\left(\underline{\underline{R}}_C\right)$$

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Full CSIT Tx-precoder Design:

$$d^2 trace\left(\underline{\underline{R}}_H . \underline{\underline{B}} . \underline{\underline{B}}^H\right)\Big|_{MAX}$$

$$s.t. \quad E_s . trace\left(\underline{\underline{B}} . \underline{\underline{B}}^H\right) = E_T$$

Since, for definite positive matrixes, we have:

$$Trace\left(\underline{\underline{F}} . \underline{\underline{G}}\right) \leq \lambda_{\max}\left(\underline{\underline{F}}\right) . Trace\left(\underline{\underline{G}}\right)$$

Equal when:

$$\underline{\underline{B}} = E_T . \underline{e}_{\max} . \underline{cs}^H \qquad \forall \underline{cs} \text{ unit norm}$$

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Full CSIT Architecture and Performance

$$P_e = Q\left(\sqrt{\frac{E_T}{N_0}.\lambda_{MAX}\left(\underline{\underline{R}}_H\right).\frac{3}{2^{n_s}-1}}\right)$$

$$\underline{\underline{H}} = \underline{\underline{V}}_H.\underline{\underline{L}}.\underline{\underline{U}}^H$$

Receiver: $\underline{\underline{V}}_H$

Channel gain: $\underline{\underline{L}}$

Transmiter: $\underline{\underline{U}}^H$

$$\underline{\underline{A}} = \underline{\underline{H}}.\underline{\underline{B}} = \left(\sum_{r=1}^{\min(n_T,n_R)} \lambda_r.\underline{v}_r.\underline{u}_r^H\right).E_T.\underline{u}_{\max} = \lambda_{\max}.E_T.\underline{v}_{\max}$$

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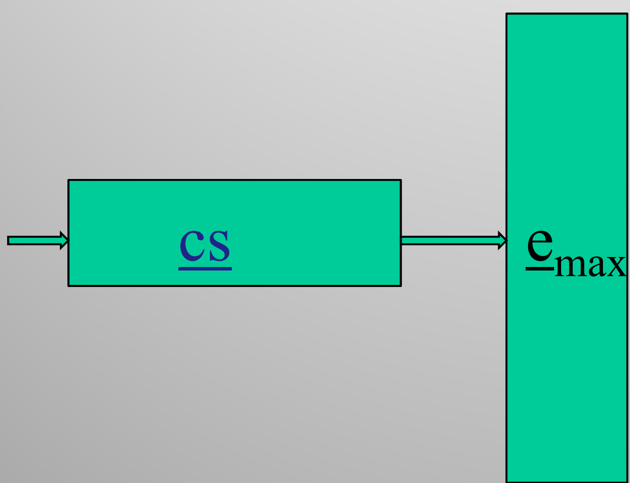
The maximum of the SNR results when ….

$$\underline{\underline{B}} = \underline{e}_{max}\,\underline{cs}^{H}$$

Where $\underline{e}_{max}$ is the maximum eigenvector of the channel matrix



This is a spreading vector which allows multiple access to the receiver. It has to be norm one in order to do not alter the transmitted power.

Note that for maximum SNR the space time processing at Tx reduces to a single beamformer (sometimes called beamforming solution)

Note that $$\underline{\underline{R}}_C \underline{\underline{E}} = \underline{\underline{D}}\underline{\underline{E}}$$

$$\underline{\underline{R}}_C = \underline{\underline{E}}\underline{\underline{D}}\underline{\underline{E}}^H = \sum_{q=1}^{n_0} \lambda_q \underline{e}_q \underline{e}_q^H$$

Or, using the svd of the equivalent channel

$$\underline{\underline{R}}_0^{-1/2} \underline{\underline{H}} = \underline{\underline{V}}\underline{\underline{D}}^{1/2} \underline{\underline{U}}^H \quad \Rightarrow \quad \begin{cases} \underline{\underline{B}} = \underline{\underline{U}} \\ \underline{\underline{A}} = \underline{\underline{R}}_0^{-1/2} \underline{\underline{V}} \end{cases}$$

This is equivalent to say that B and A diagonalice the channel setting their values to U and V respectively. Each resulting channel, in number equal to the rank of the channel matrix, has a gain equal to the eigenvalue. THEN; THE BEST POLICY IS TO SEND ALL THE AVAILABLE POWER TROUGH THE BEST CHANNEL

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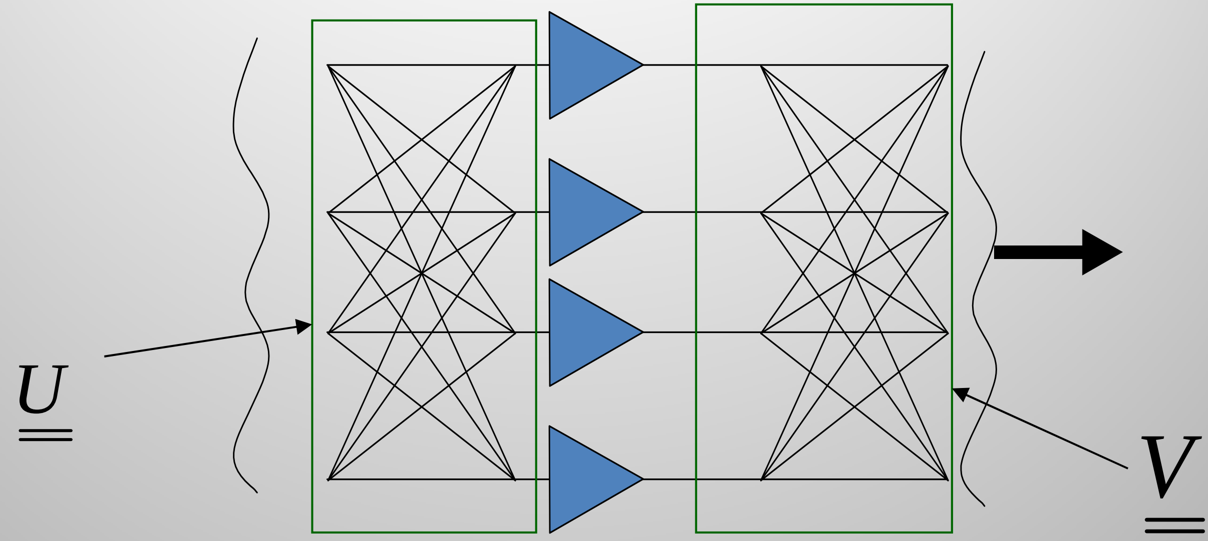
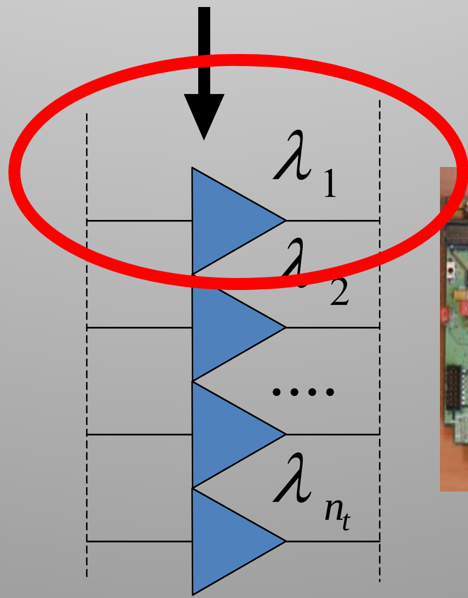
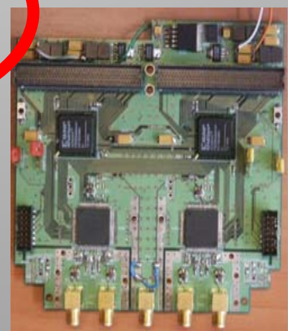
Intuitive Explanation





Just use the best channel for all the available Tx power



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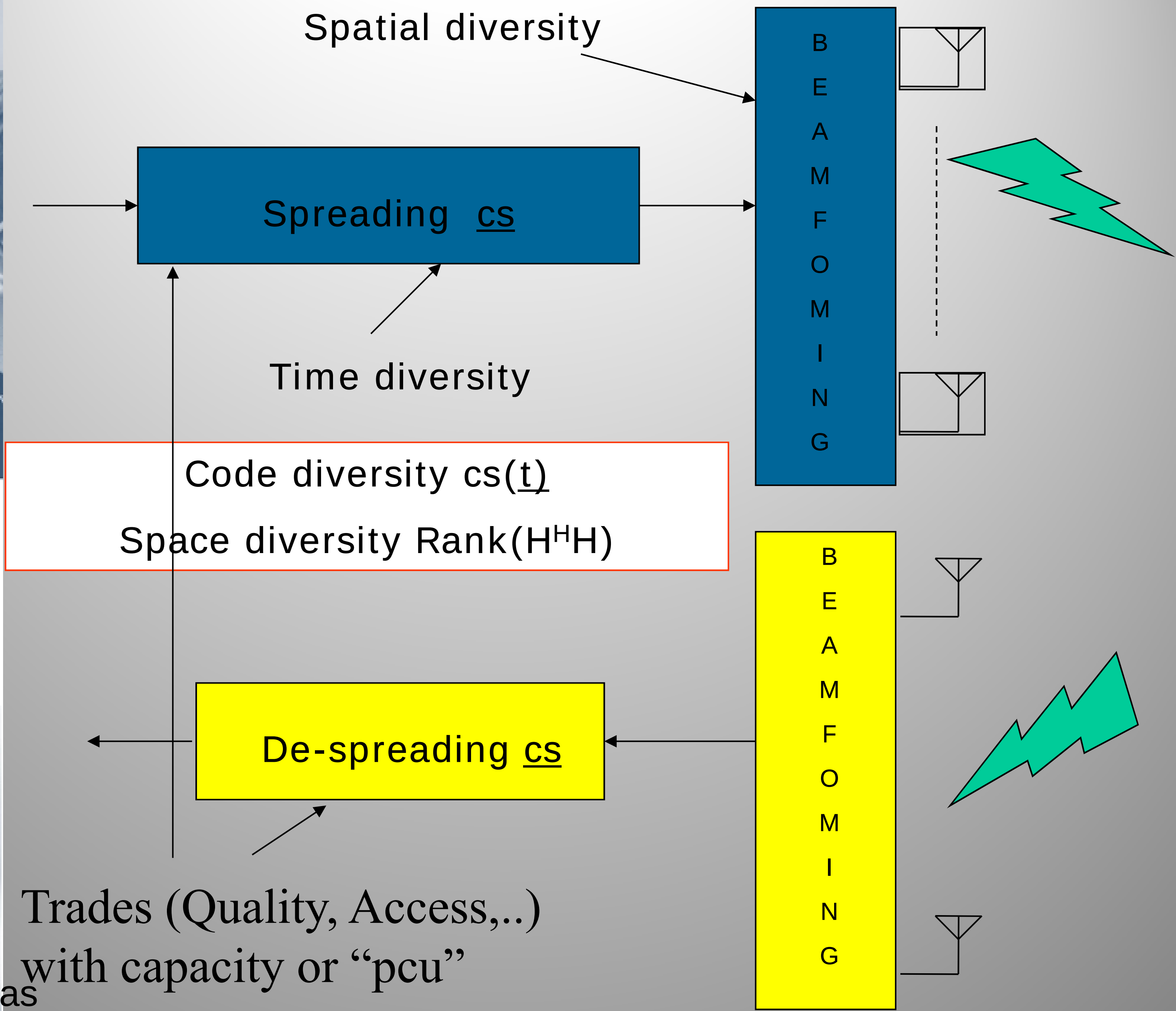

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CSIR Only (Define a Game)

Define PLAYERS	TX Engineer	Channel
Define payoffs	Minimize P_E	Destroy Rx Energy
Define strategy of the game	Channel plays first, Tx engineer plays second. Only one pass	
Constrains/player	E_T	$Trace\left(\underline{\underline{R}}_H\right) \geq \rho$

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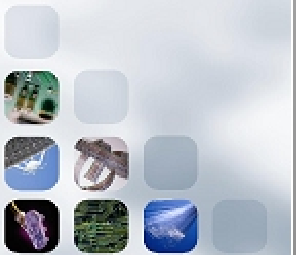

CSIR Only

$$\max_{\underline{\underline{Q}}}\left(\min_{\underline{\underline{R}}_h}\left(\underline{\underline{R}}_H.\underline{\underline{Q}}\right)\right)$$

One play game

$$\underline{\underline{R}}_H = \underline{\underline{H}}^H.\underline{\underline{R}}_0^{-1}.\underline{\underline{H}} \qquad \underline{\underline{Q}} = \underline{\underline{B}}.\underline{\underline{B}}^H$$

$$Traza\left(\underline{\underline{R}}_H.\underline{\underline{B}}.\underline{\underline{B}}^H\right) \geq \lambda_{\min}\left(\underline{\underline{B}}.\underline{\underline{B}}^H\right).Traza\left(\underline{\underline{R}}_H\right)$$

Channel plays

Tx Eng. plays as maximize the minimum eigenvalue when the trace is constrained

$$\underline{\underline{B}} = \left(\frac{E_T}{n_T}\right)^{1/2}.\underline{\underline{\Pi}} \qquad with \quad \underline{\underline{\Pi}}.\underline{\underline{\Pi}}^H = \underline{\underline{I}}_{n_T}$$

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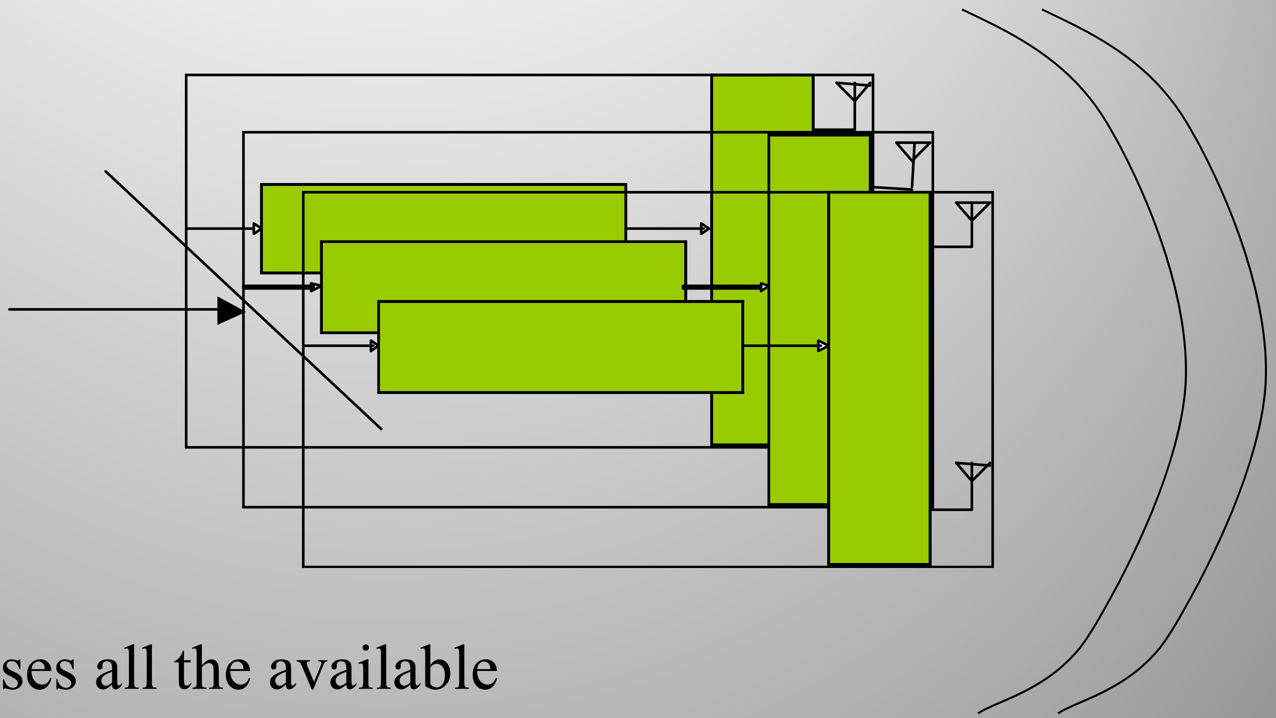
The CSIR Performance

$$\underline{\underline{B}} = \underline{\underline{\Pi}}.\sqrt{\frac{k_2}{n_T}} \quad y \quad \gamma = 4E_s.Traza\left(\underline{\underline{R}}_H.\underline{\underline{B}}.\underline{\underline{B}}^H\right) = \frac{4.E_s.k_2}{n_T}.Traza\left(\underline{\underline{R}}_H\right)$$

$$P_e^{NO-CSI} = Q\left(\sqrt{\frac{E_T}{N_0}.\left(\frac{Traza\left(\underline{\underline{R}}_H\right)}{n_T}\right).\left(\frac{3}{\left(2^{n_S}-1\right)}\right)}\right)$$

We pass from the maximum eigenvalue gain to the arithmetic mean of the eigenvalues.

The CSIR only Strategy



Uses all the available eigenmodes

UPA

Uniform Power Allocation

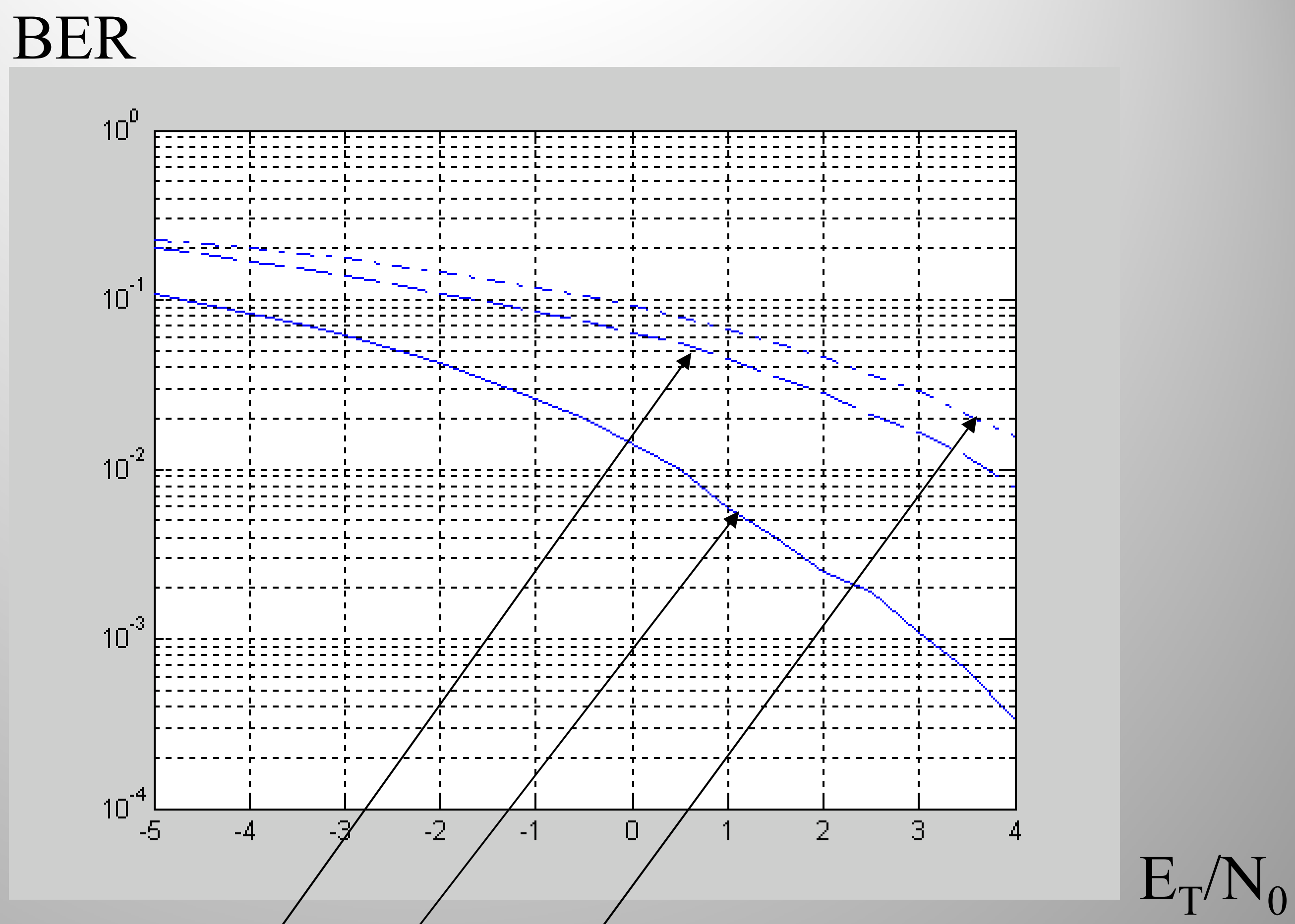
PCU and Increase Rate

Since $\underline{\underline{B}}$ Is unitary, it has to be full rank, i.e. Size n_T by n_T (at least)

One symbol for n_T channel uses → Rate $1/n_T$

It is possible to increase this rate??

SPACE-TIME BLOCK CODES



BER versus SNR for a MIMO system without CSIT. De abajo a arriba: MIMO(4,4) six streams and four PCUs (r=3/4); MIMO(3,3); MIMO(2,2) four streams and 2 PCUs (r=1).

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