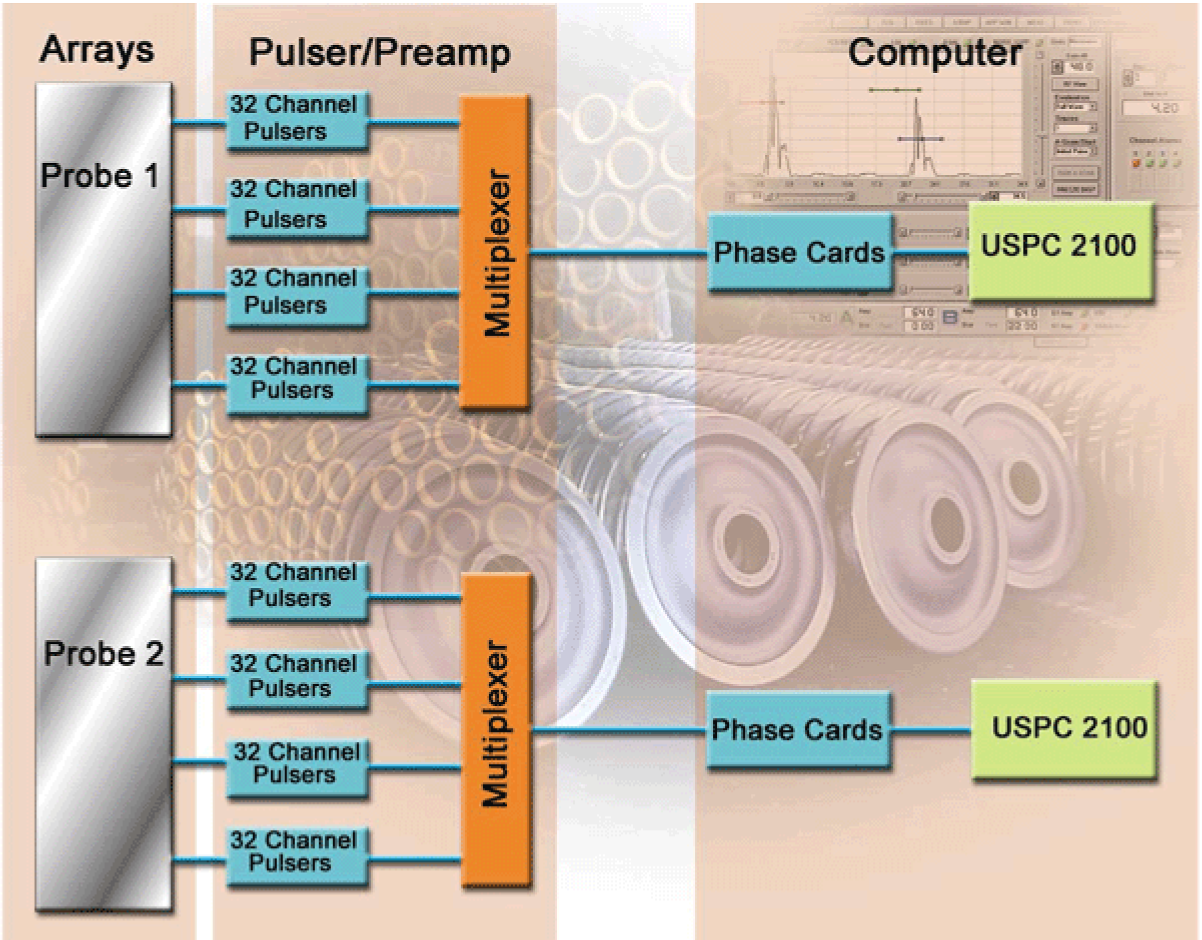
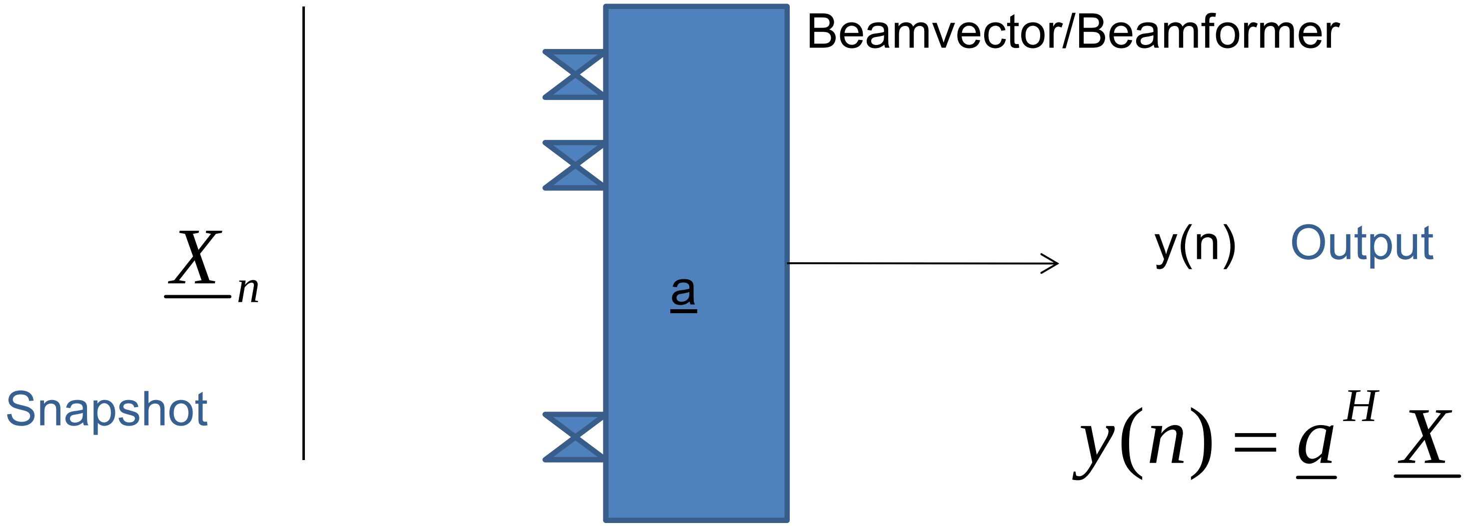
BEAMFORMING



Beamformer/Beamvector



$$y(n) = \underline{a}^H \underline{X}_n$$

Beamformer response to an unitary point source, located in the far field and mono-chromatic impinging the aperture from a given elevation and azimuth.

$$y_{\theta,\phi}(n) = \underline{a}^H \underline{S}$$

$$s(q) = \exp\left(j2\pi . f \frac{\sin(\theta)}{c} d_q . \cos(\phi_q - \phi) \right)$$

The so-called ARRAY FACTOR is: $G(\theta,\phi) = \left| \underline{a}^H \underline{S} \right|^2$

The array factor is closely realted with the DIRECTIVITY of the aperture

$$D(\theta,\phi) = \frac{\text{Power at the direction/squaremt.}}{\text{Isotropic power density}} = \frac{G(\theta,\phi)}{\underline{a}^H \underline{a}} \left(\frac{A_{effective}}{4\pi . r^2} \right)^{-1}$$

This directivity depends on the effective area of the receiver the location and with the norm of the beamformer, which multiplied by the applied signal power provides the global transmitted power, and the array factor.

Do not confues D or G with the antenna GAIN. The antenna gain (always is a loss) equals to one when ther is not mutual coupling between aperture elements

The transmitted power Pt is always less than or equal (no MC) than the radiated power.

ANTENNA GAIN

Being vector v the vector containing the signals applied to each antenna, MC (Mutual Coupling) implies that the radiated vector x is related to the original by means of the MC matriz Z

$$\underline{x} = \underline{\underline{Z}}\,\underline{v}$$

$$\text{where} \quad tr\left(\underline{\underline{Z}}^H \underline{\underline{Z}}\right) \le Q$$

$$\text{and} \quad E\left(\underline{v}\,\underline{v}^H\right) \equiv \underline{\underline{R}}_v = \frac{P_T}{Q}\underline{\underline{I}}$$

Being Q the total number of antennas

Since the radiating power is given by:

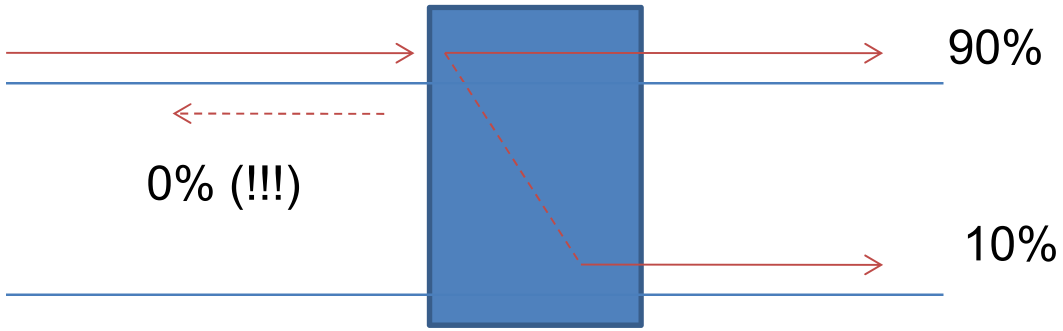
$$P_R = tr\left(E\left(\underline{x}\,\underline{x}^H\right)\right) = tr\left(\underline{\underline{Z}}E\left(\underline{v}\,\underline{v}^H\right)\underline{\underline{Z}}^H\right) \le \lambda_{\max}\left(\underline{\underline{R}}_v\right) tr\left(\underline{\underline{Z}}\,\underline{\underline{Z}}^H\right) =$$

$$= \left|\begin{matrix}\text{Uniform and independent} \\ \text{power allocation}\end{matrix}\right| = \frac{P_T}{Q} tr\left(\underline{\underline{Z}}\,\underline{\underline{Z}}^H\right) \le P_T$$

Example of the MC power loss:

$$\underline{\underline{Z}} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad tr\left(\underline{\underline{Z}}\,\underline{\underline{Z}}^H\right) = 2 \quad \frac{P_{Rad}}{P_T} = 1 \quad 100\%$$

$$\underline{\underline{Z}} = \begin{pmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{pmatrix} \quad tr\left(\underline{\underline{Z}}\,\underline{\underline{Z}}^H\right) = 1.64 \quad \frac{P_{Rad}}{P_T} = 0.82 \quad 82\%$$



THE ARRAY RESPONSE

The receiver snapshot for desired interferers and front-end noise is:

$$\underline{X}_n = a_d(n).\underline{S}_d + \sum_{i=1}^{NI} a_i(n).\underline{S}_i + \underline{w}_n$$

The response of a beamformer defined as vector A is:

$$y(n) = \underline{A}^H.\underline{X}_n = a_d(n).\underline{A}^H\underline{S}_d + \sum_{i=1}^{NI} a_i(n).\underline{A}^H.\underline{S}_i + \underline{A}^H\underline{w}_n$$

Using the power of the beamformer's output

$$P_y = E\left[|y(n)|^2\right]$$

$$P_y = \underline{A}^H.E\left[\underline{X}_n.\underline{X}_n^H\right].\underline{A} = \underline{A}^H.\underline{\underline{R}}.\underline{A}$$

Together with the covariance of the received snapshot……

$$\underline{\underline{R}} = P_d.\underline{S}_d.\underline{S}_d^H + \sum_{i=1}^{NI} P_i.\underline{S}_i.\underline{S}_i^H + \sigma^2.\underline{\underline{I}}$$

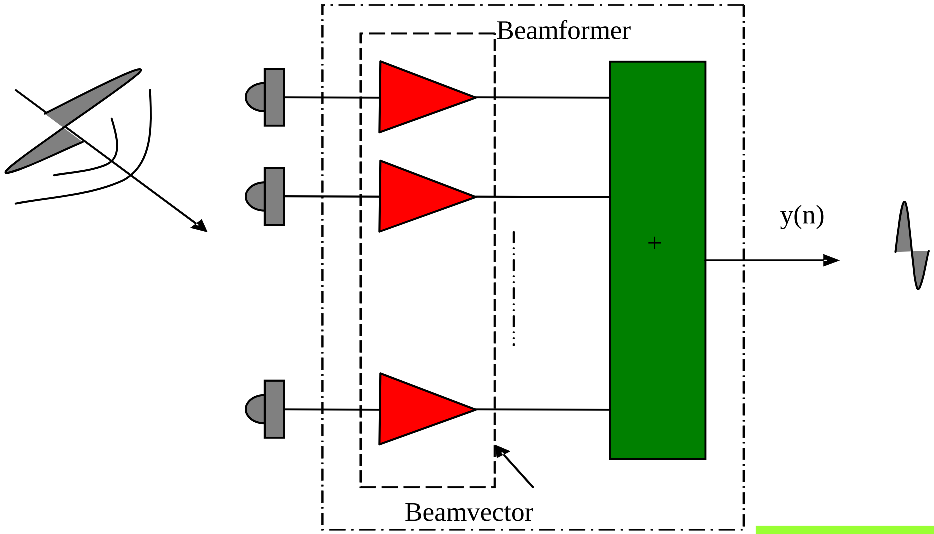
The different contributions at the array output is:

$$P_y = P_{señal} + P_{\text{int}\, erferencias+ruido} =$$

$$= P_d \left| \underline{A}^H . \underline{S}_d \right|^2 + \sum_{i=1}^{NI} P_i \cdot \left| \underline{A}^H . \underline{S}_i \right|^2 + \sigma^2 . \left\| \underline{A} \right\|^2$$

Our objectives or goals are:

- Set to 1 (0 dB.) AGC Automatic Gain Control
- Set to zero or minimize interferers comntributions
- Minimize norm or minimize response to undirectional noise or minimiza the area of the beam response



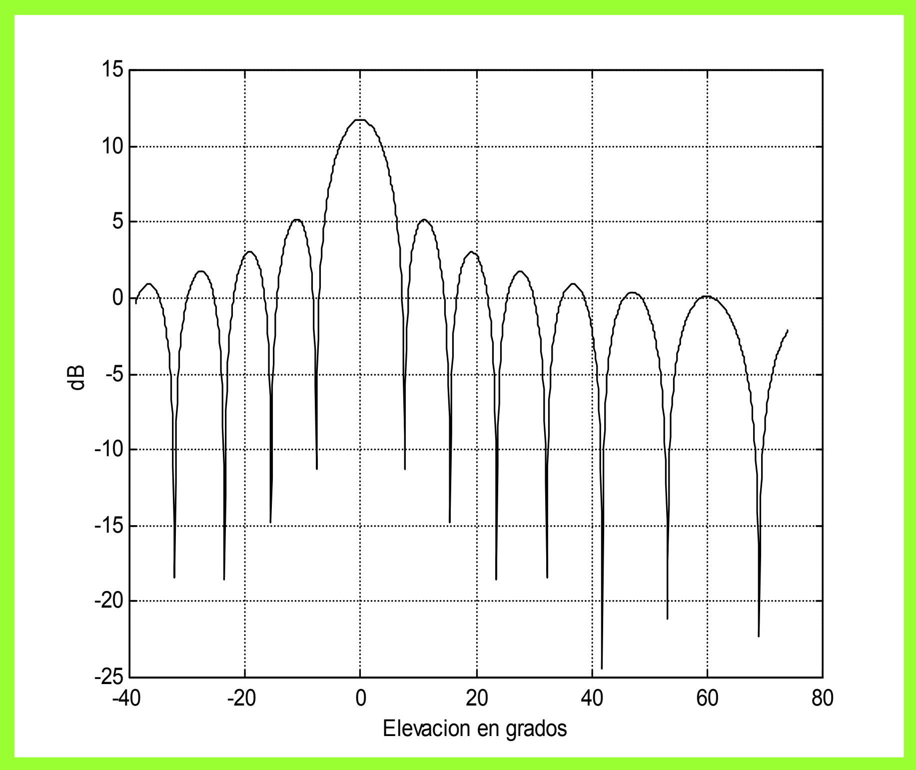
The phased array

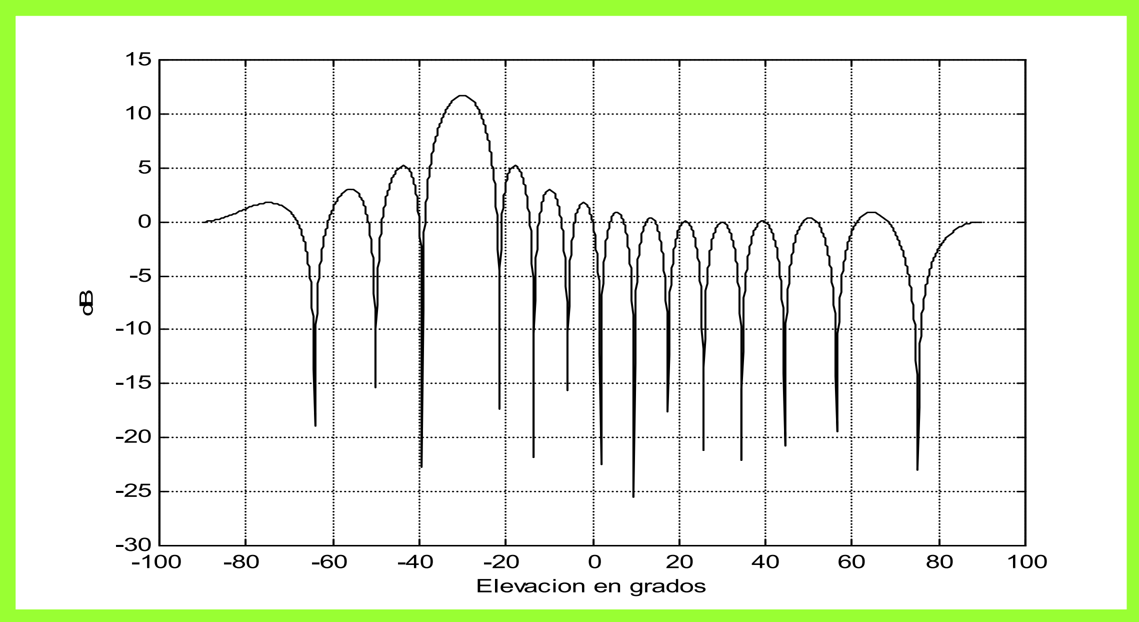
$$\underline{a} = \underline{S}_d$$

$$G(\theta, \phi) = \underline{A}^H . \underline{S}$$

Beamvector

Steering



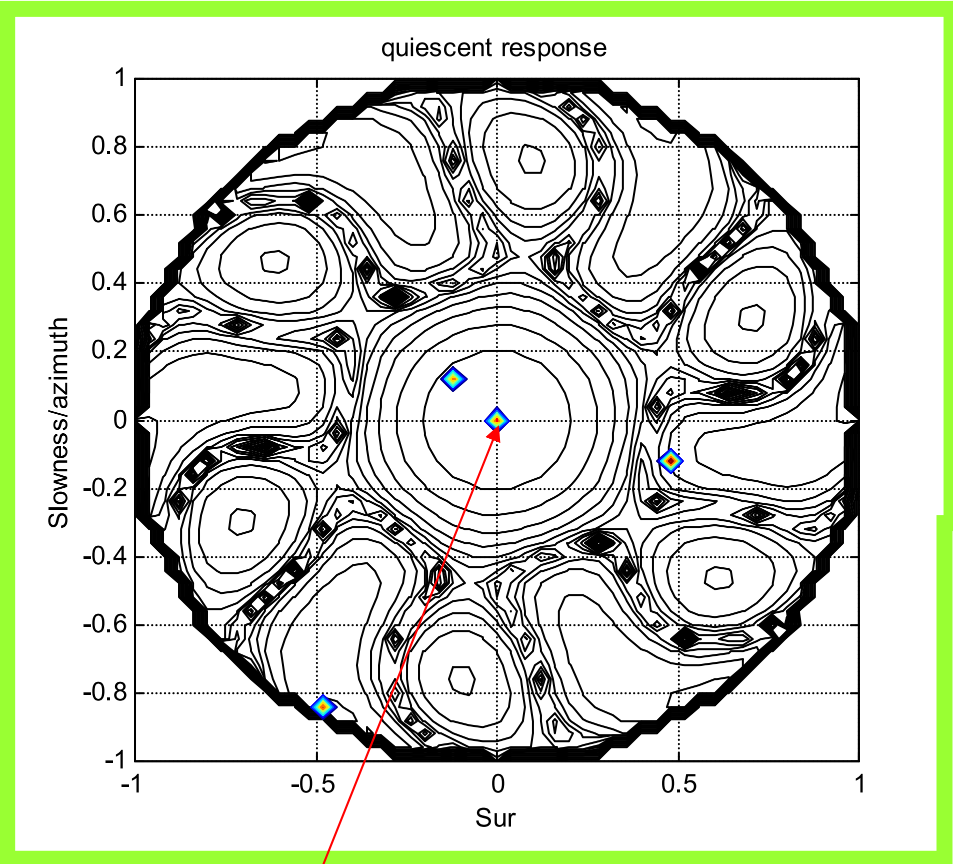


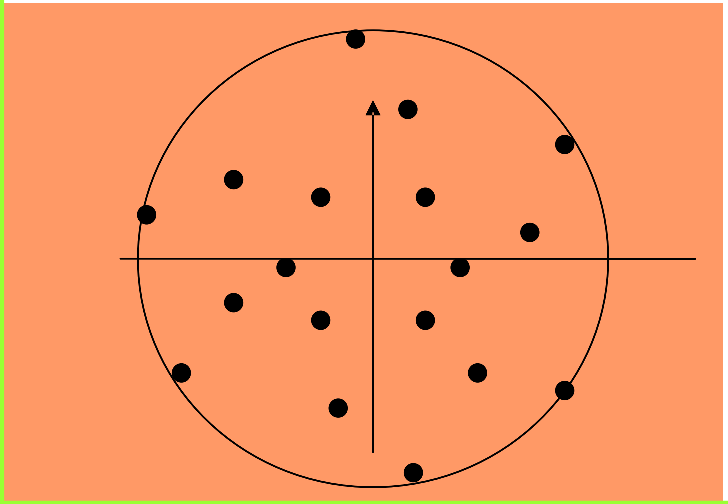
-30 degrees

$$\underline{A} = \underline{S} = \begin{bmatrix} \vdots \\ \exp(-j2\pi . f_o \dfrac{\sin(\theta)}{c} . d_q) \\ \vdots \end{bmatrix}$$

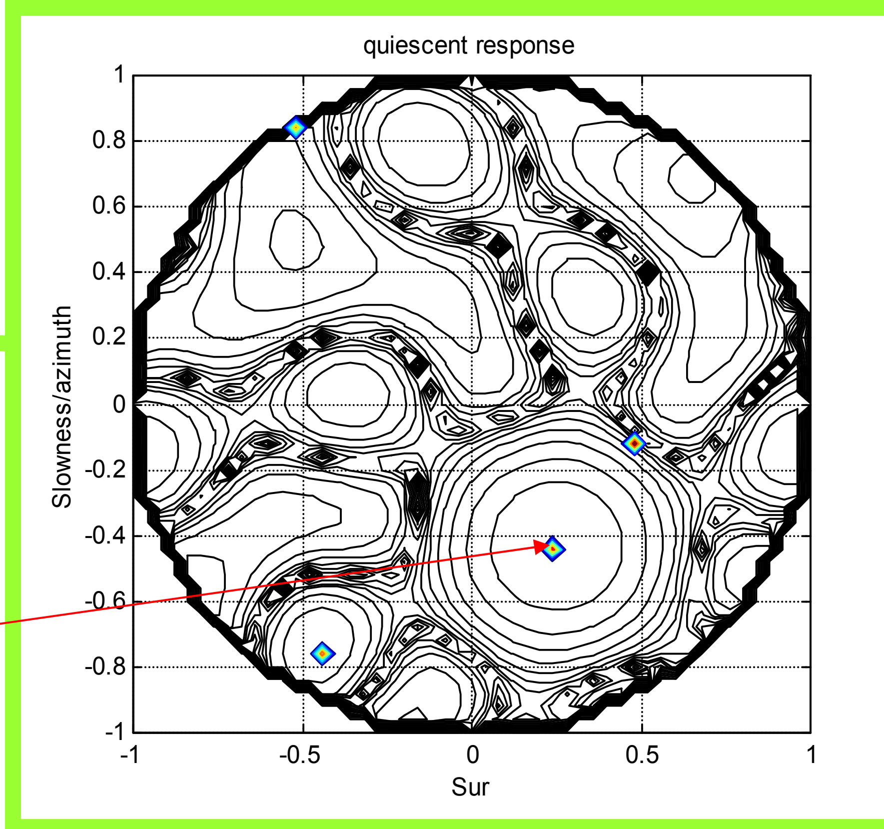
ULA Array

$$d_q = q . \frac{\lambda}{2}$$



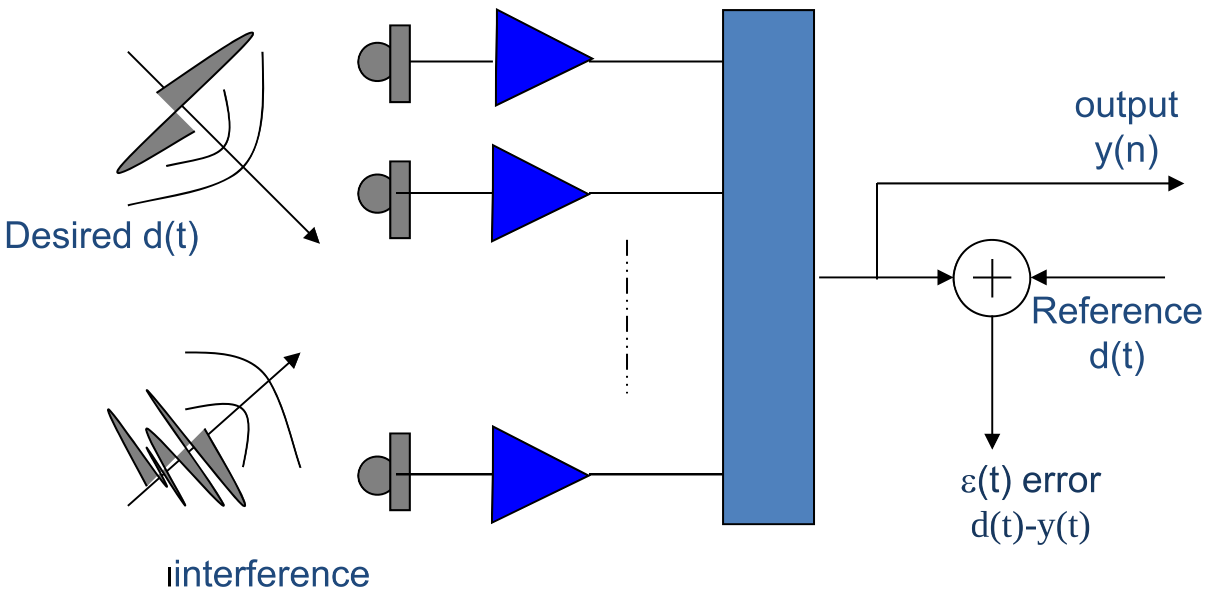


Steered to
0º elevation



Steered to 30º of
elevation and 150º of
azimuth

TIME REFERENCE BEAMFORMING



The MSE at the output is

$$\xi = E\left[\varepsilon(n).\varepsilon^*(n)\right] = E\left[\left|d(n) - \underline{A}^H.\underline{X}_n\right|^2\right]$$

And, taking expected values

$$\xi = P_d + \underline{A}^H.\underline{\underline{R}}.\underline{A} - \underline{P}^H.\underline{A} - \underline{A}^H.\underline{P}$$

where $P_d = E\left(d(n)d^*(n)\right)$ The power of the desired or reference

$\underline{\underline{R}} = E\left(\underline{X}_n \underline{X}_n^H\right)$ The covariance matrix of the received snapshots

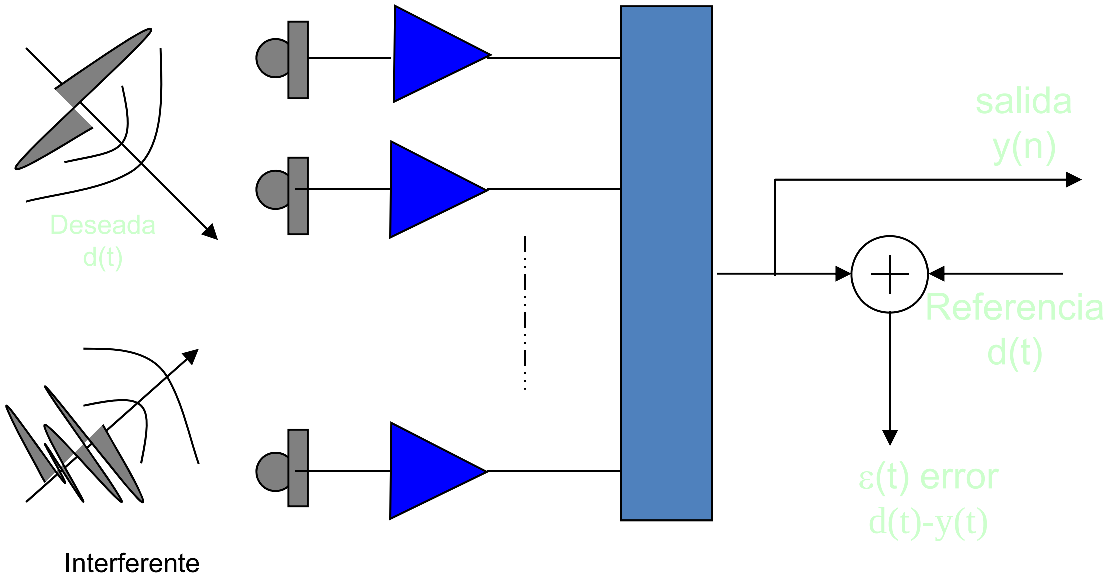
$\underline{P} = E\left(\underline{X}_n d^*(n)\right)$ The P-vector or cross correlation between snapshots and the reference.

The gradient of the MSE with respect the conjugate of the desired beamvector is:

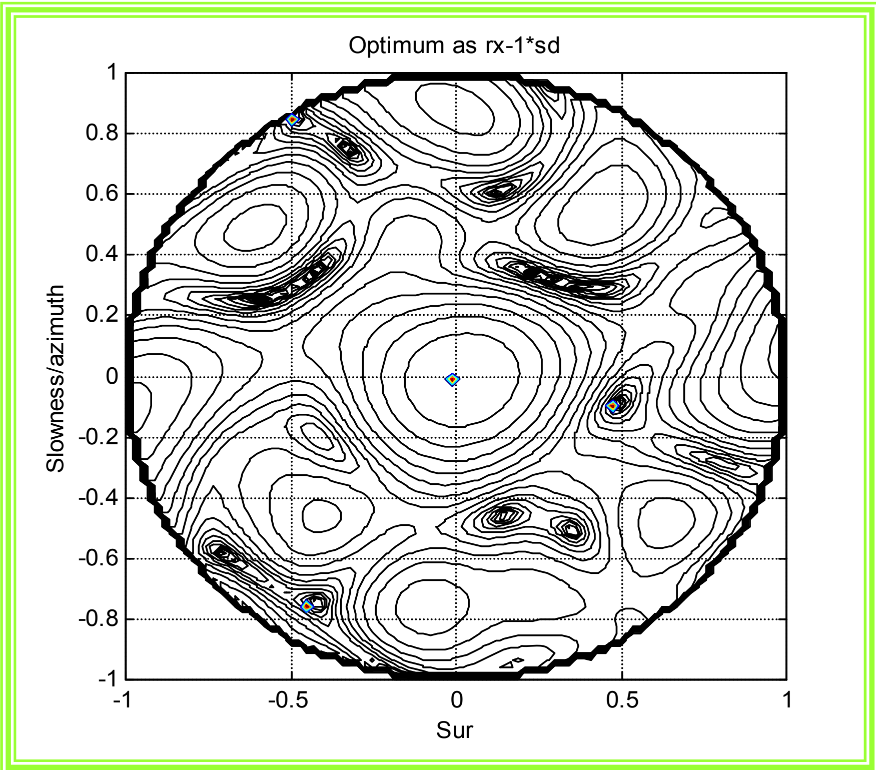
$$\nabla_{\underline{A}^H} \xi = \underline{\underline{R}}.\underline{A} - \underline{P} = \underline{0}$$

This provides the optimum (MSE sense) Time Reference Beamformer

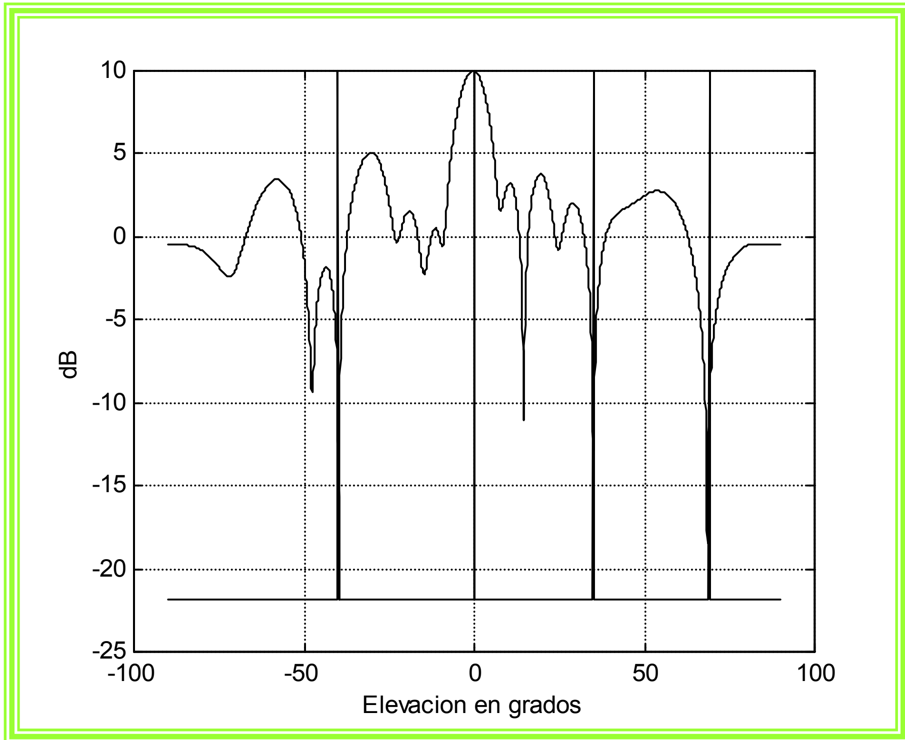
$$\underline{A}_{opt} = \underline{\underline{R}}^{-1}.\underline{P}$$



Deseada
d(t)
salida
y(n)
Referencia
d(t)
ε(t) error
d(t)-y(t)
Interferente



Optimum as rx-1*sd
Slowness/azimuth
Sur



dB
Elevacion en grados

MSE Optimum SNR design

The desired covariance matrix $\underline{\underline{R}}_d = P_d \underline{S}_d \underline{S}_d^H$

The noise plus interference Covarinace matrix $\underline{\underline{R}} - \underline{\underline{R}}_d = \underline{\underline{R}}_I + \sigma^2 \underline{\underline{I}}$

SNR at the beamformer's output $SNR = \dfrac{\underline{A}^H \underline{\underline{R}}_d \underline{A}}{\underline{A}^H (\underline{\underline{R}} - \underline{\underline{R}}_d) \underline{A}}$

The solution of this quotient of two cuadratic forms is (setting constant the numerator/denominator minimice/maximice the denominator/numerator. This results in a generalized eigenvalue problem. The solution is the corresponding eigenvector

$$\underline{\underline{R}}_d \underline{A} = \lambda_{\max} \left(\underline{\underline{R}} - \underline{\underline{R}}_d \right) \underline{A}$$

When the covariance of the desired is rank one, this solution coincides with the MSE we derived before

$$(\lambda_{\max} + 1) \underline{\underline{R}}_d \underline{A} = \lambda_{\max} \underline{\underline{R}} \underline{A}$$

$$\Rightarrow \underline{A} \propto \underline{\underline{R}}^{-1} \underline{S}_d$$

Note that $\underline{P} = P_d \cdot \underline{S}_d$

Thus the optimum MSE can be written as:

$$\underline{A} = \underline{\underline{R}}^{-1}\underline{P} \propto \underline{\underline{R}}^{-1}\underline{S}_d$$

This last version of the optimum beamformer reveals that using a temporal reference is equivalent to have the corresponding DOA of the desired to form the beamformer. Thus, temporal reference is equivalent to spatial reference. Furthermore both maximices the SNR.

Interestingly, there is also another formula for the optimum beamformer. Note that naming with matrix Ro the interference plus noise covariance, again for rank one desired, the optimu solution can be written as:

$$\underline{\underline{R}}_d \underline{A} = \lambda_{max}\left(\underline{\underline{R}} - \underline{\underline{R}}_d\right)\underline{A}$$

$$= \lambda_{max}\underline{\underline{R}}_0 \underline{A}$$

$$\underline{A} \propto \underline{\underline{R}}_0^{-1}\underline{S}_d$$

¡¡¡3 DIFFERENT MANNERS TO FORMULATE THE OPTIMUM BEAMFORMER!!!

The relationship between the MSE and the SNR is the following:

Since the error and the snapshot components are uncorrelated, the error and the output also are uncorrelated, thus:

$$\varepsilon(n) = d(n) - y(n) \qquad y(n) + \varepsilon(n) = d(n) \Rightarrow S + \xi_{\min} = P_{ref}$$

$$\frac{S}{\xi_{\min}} = \frac{P_d}{\xi_{\min}} - 1$$

$$SNR = \frac{P_d}{\xi_{\min}} - 1$$

This expression can be derived also from the expression of the minimum MSE:

$$\xi_{\min} = P_d - \underline{A}_{opt}^H \underline{\underline{R}}\,\underline{A}_{opt}$$

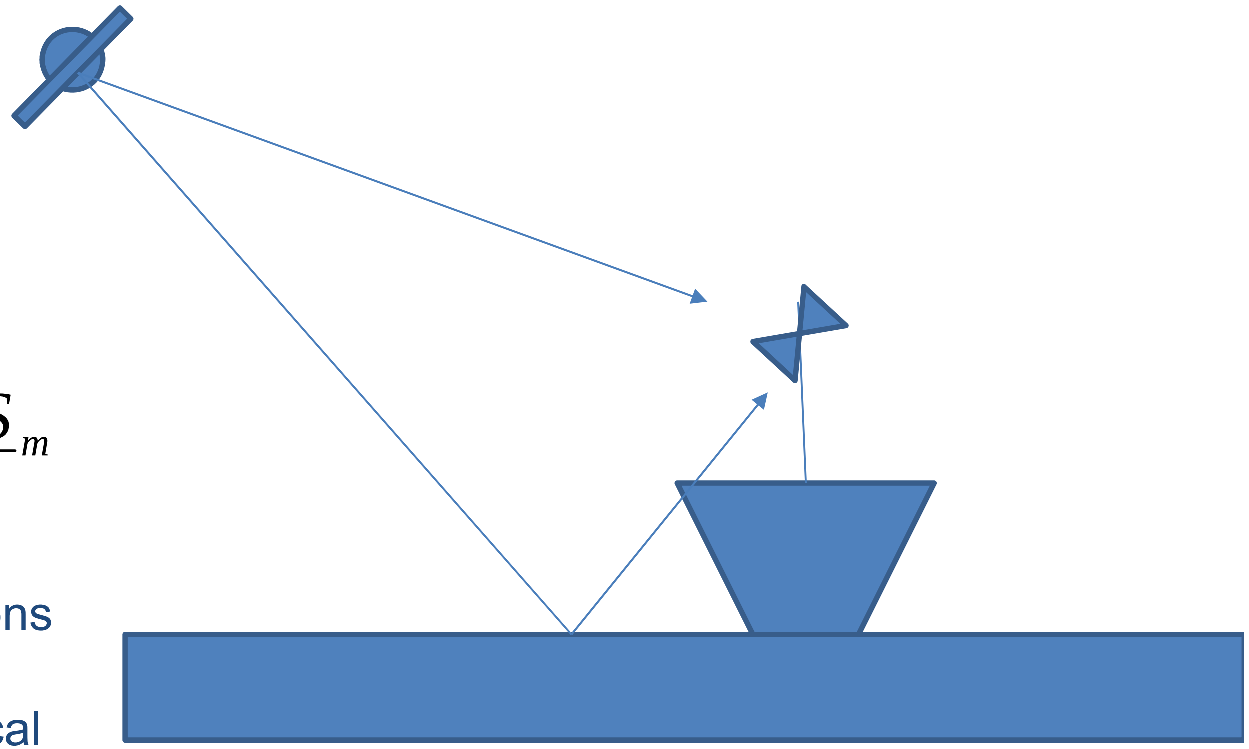
It is also interesting to write the MSE in terms of its minimum and the optimum weights (Coments: Quantization, Quadratic, Weighting by the covariance)

$$\xi = \xi_{\min} + \left(\underline{A} - \underline{A}_{opt}\right)^H \underline{\underline{R}}\left(\underline{A} - \underline{A}_{opt}\right)$$

The impact of coherent multipath

LOS and secondary arrivals produce two terms on the P-vector

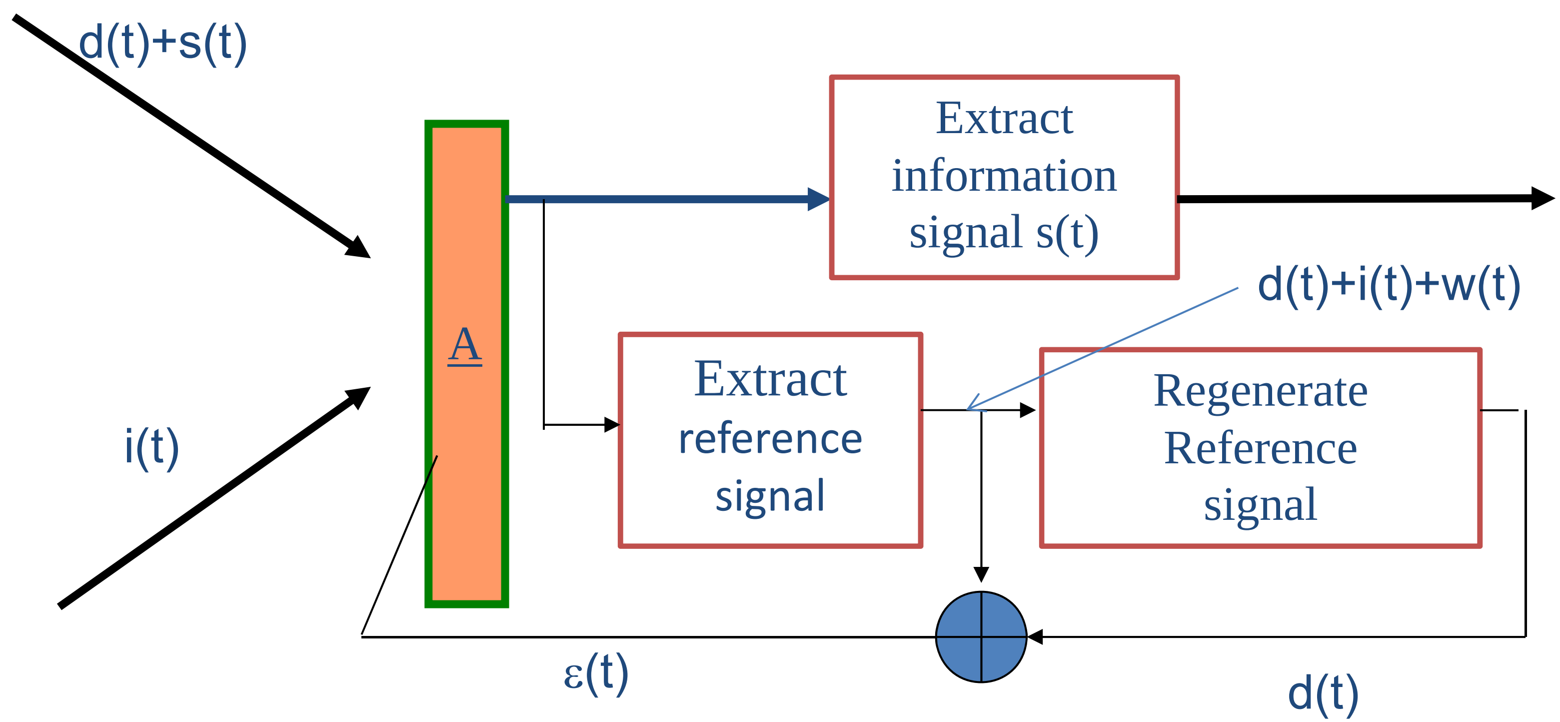
$$\underline{P} = r_d(0)\underline{S}_d + \alpha . r_d(\tau)\underline{S}_m$$



This implies that both directions are focused by the optimum beamformer. The low statistical stability of the reflection coefficient α produces a saturation of the BER when increasing the received SNR

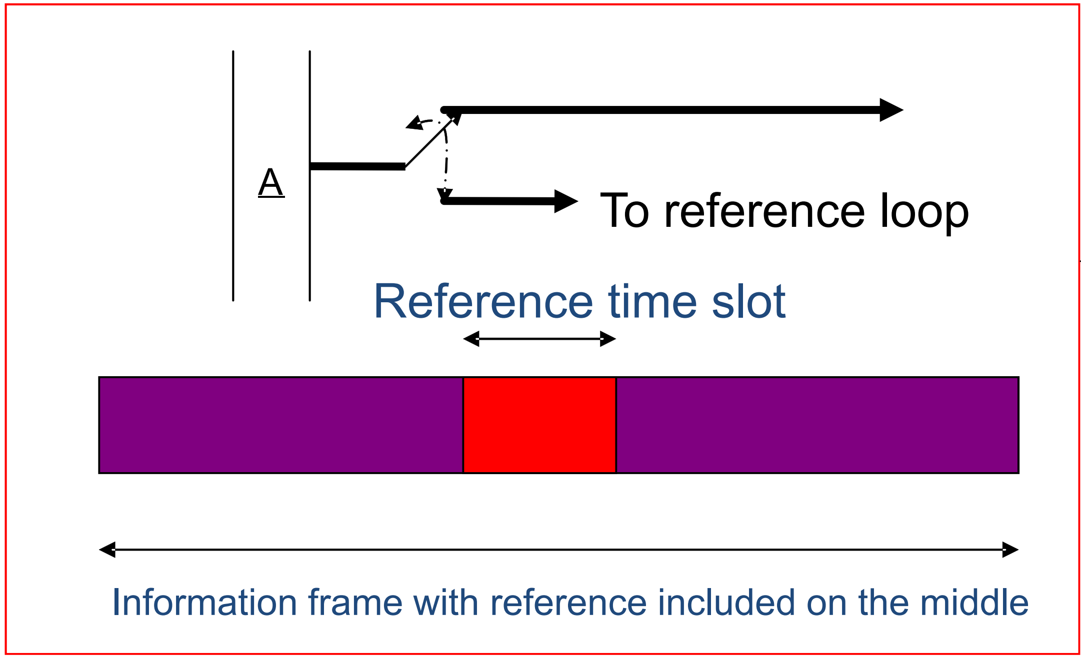
To overpass the problem:
-Spatial smoothing
-Combined temporal/spatial reference
-Polarization diversity

THE REFERENCE LOOP

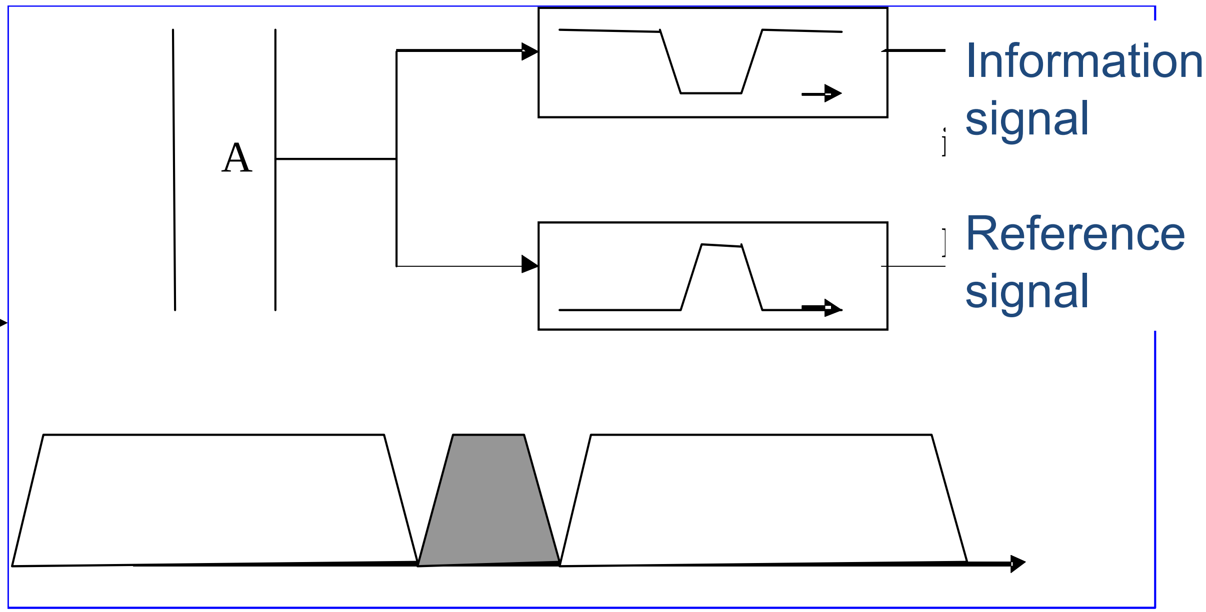


The reference d(t) is framed within the information message s(t). At the receiver, the reference is de-framed. At the output of the reference block the reference is regenerated (free of noise and interference thanks to a priori known properties). Note that the beamformer will steer the direction where d(t) is coming from.

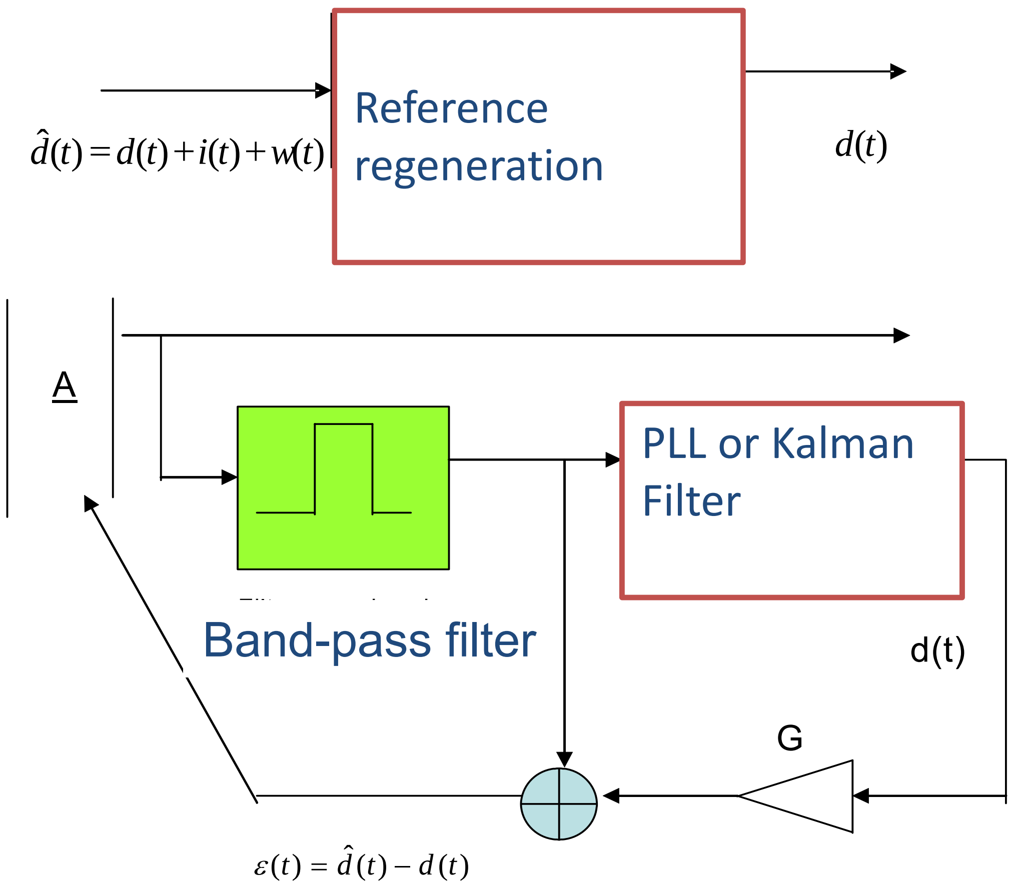
TRB- TDD



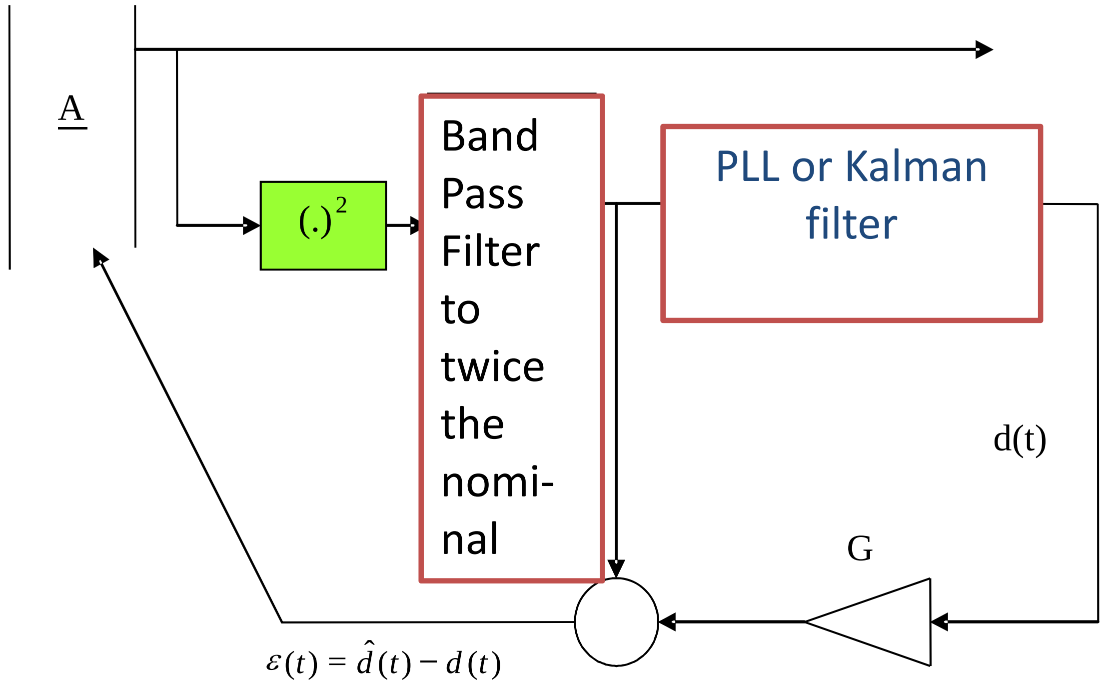
TRB-FDD



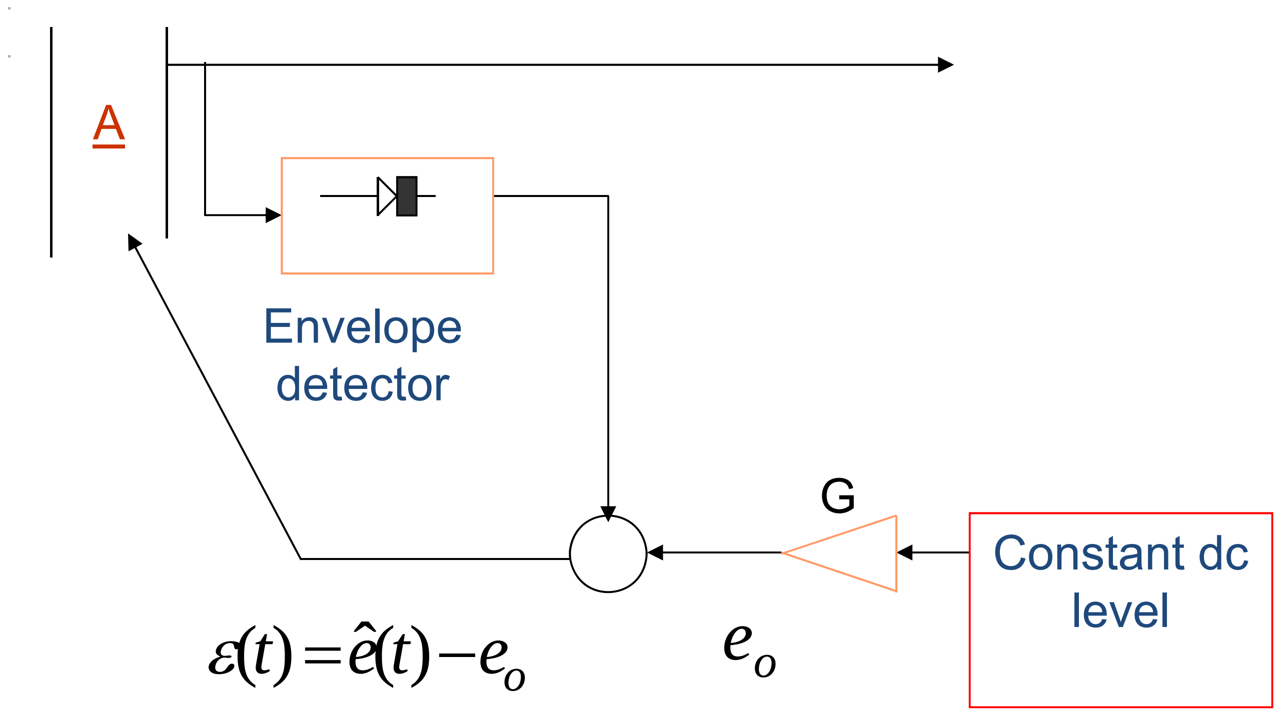
Single carrier reference loop



Un-coherent BPSK loop

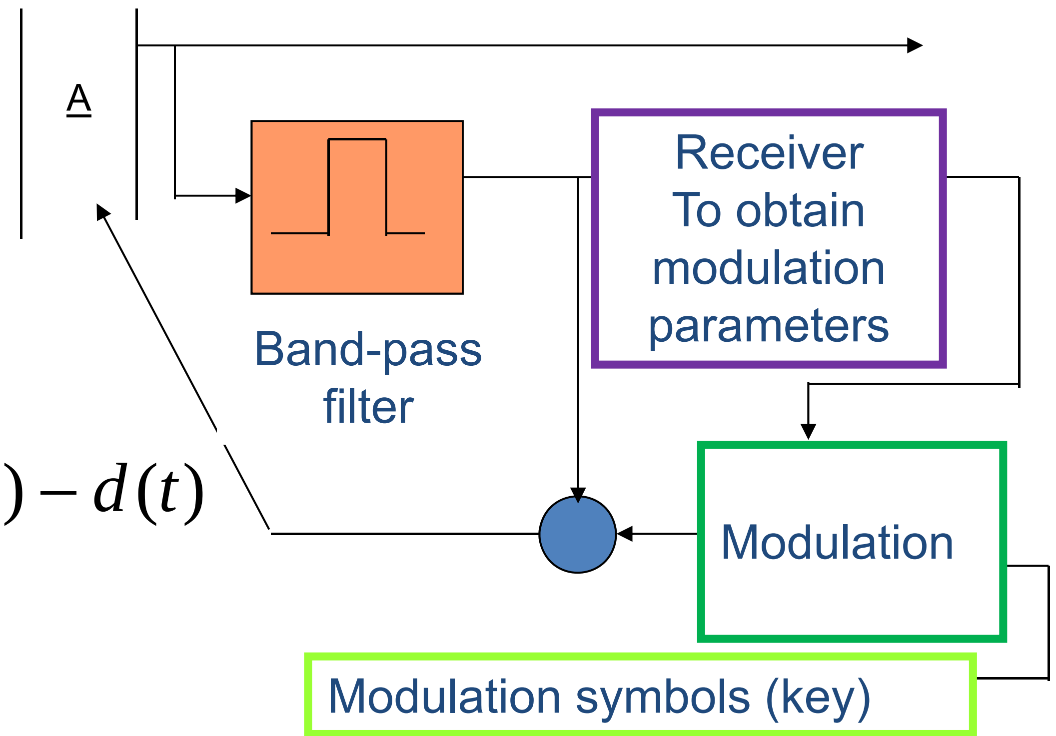


CMA-Beamformer

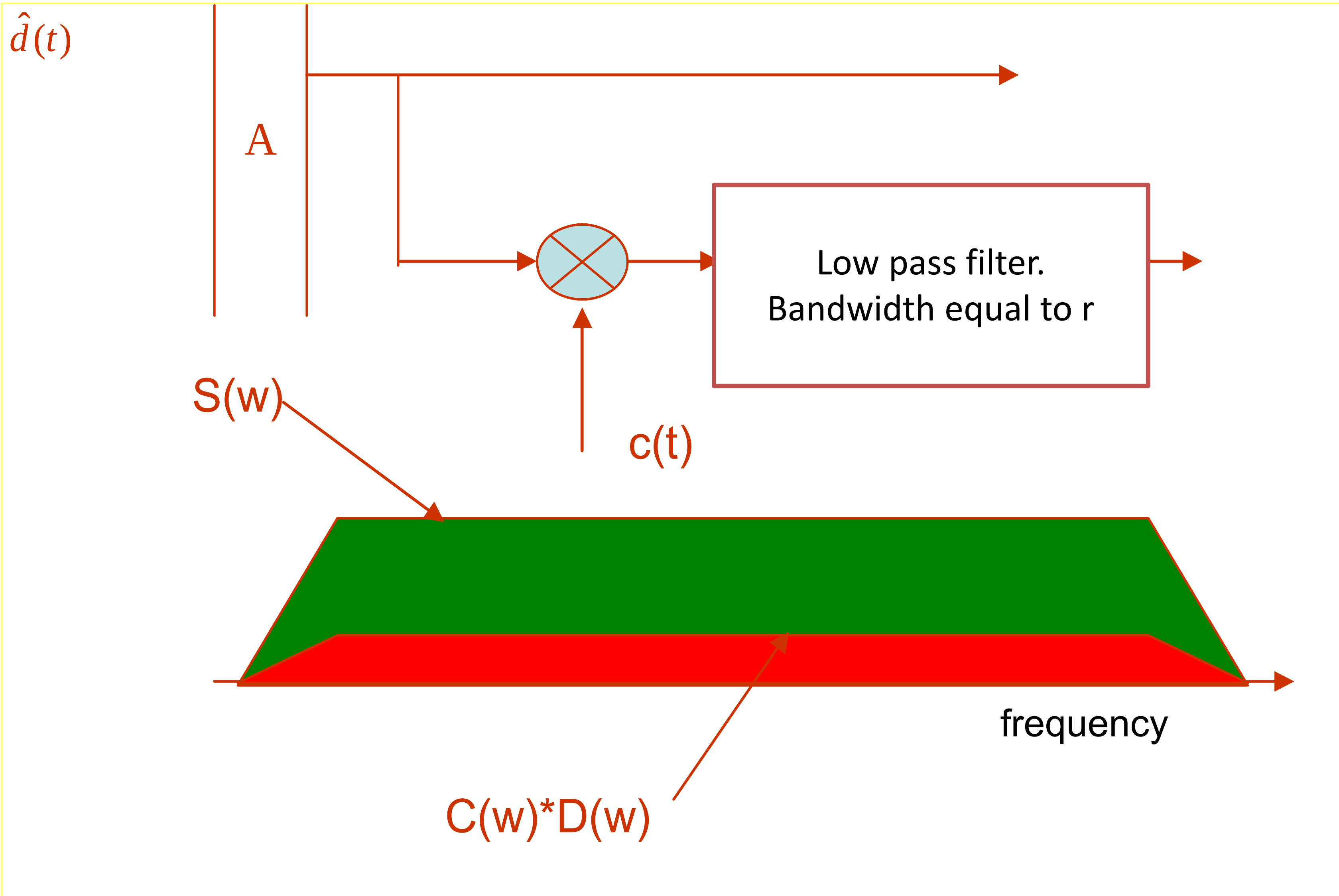


$$\varepsilon(t) = \hat{e}(t) - e_o$$

Structured reference



$$\varepsilon(t) = \hat{d}(t) - d(t)$$



$\hat{d}(t)$
A
Low pass filter.
Bandwidth equal to r
S(w)
c(t)
frequency
C(w)*D(w)

SPATIAL REFERENCE BEAMFORMING

Assuming that the steering of the desired is known, the beamformer design will be to preserve the desired response (i.e. with 0 dB.) and minimize the global output power with the previous constrain.

$$\underline{A}^H . \underline{S}_d = 1$$

$$\left. \underline{A}^H . \underline{\underline{R}} . \underline{A} \right|_{\text{minimum}}$$

The solution is:

$$\underline{A} = \frac{\underline{\underline{R}}^{-1} . \underline{S}_d}{\underline{S}_d^H . \underline{\underline{R}}^{-1} . \underline{S}_d}$$

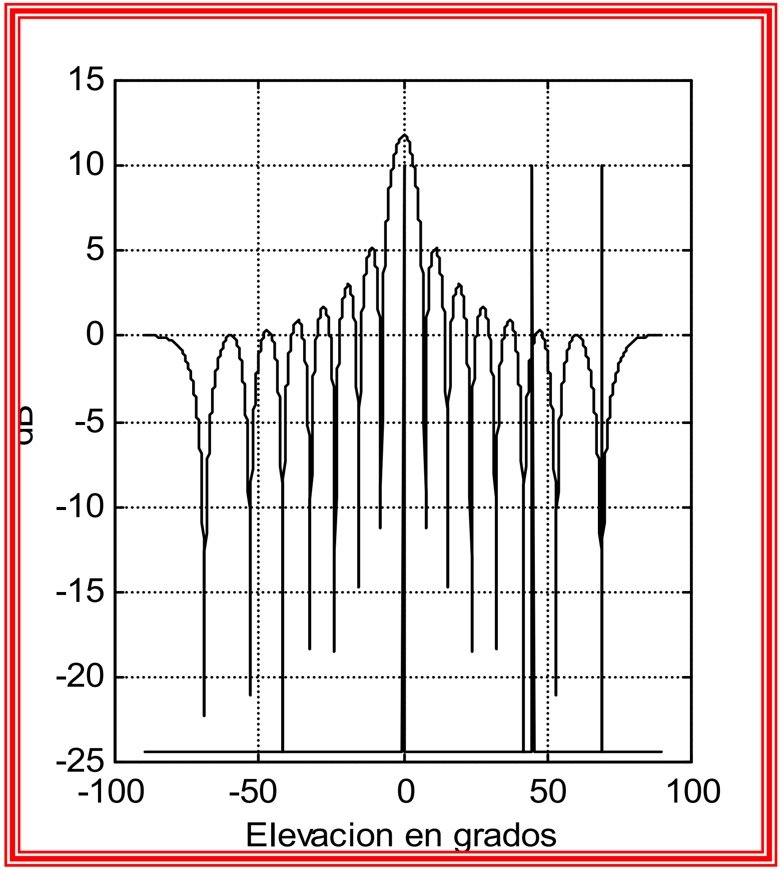
Also, assuming that the received covariance is $\underline{\underline{R}} = \alpha_d . \underline{S}_d . \underline{S}_d^H + \underline{\underline{R}}_o$
Equal to the addition of the desired plus the interference plus noise matrix, and taking into account the constrain we have:

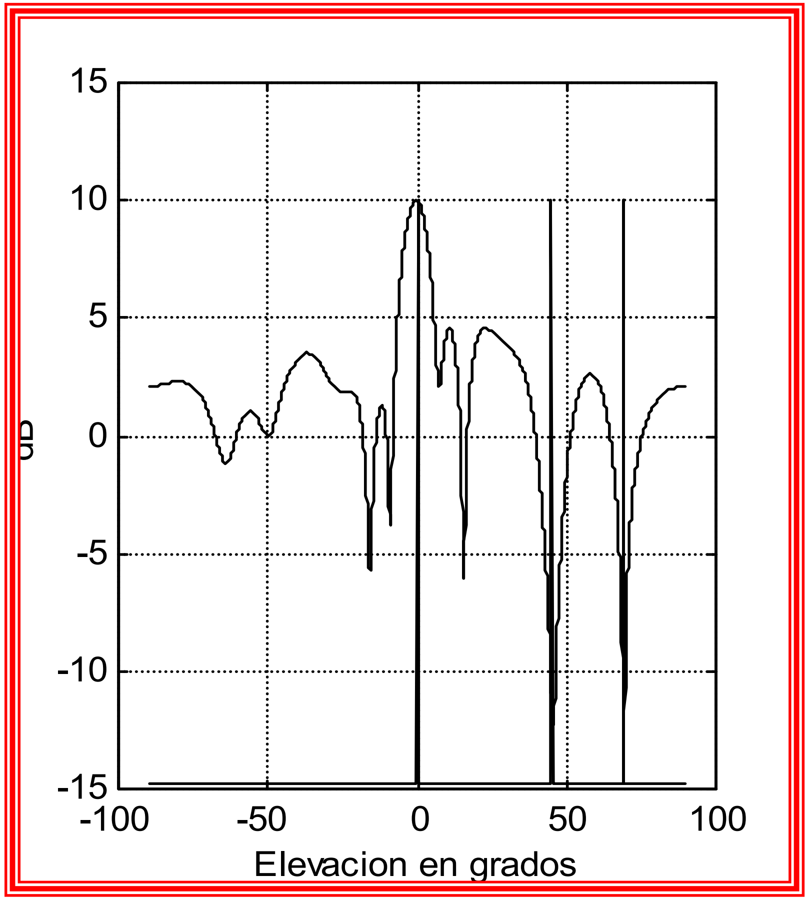
$$\underline{A}^H . \left[\alpha_d . \underline{S}_d . \underline{S}_d^H + \underline{\underline{R}}_o \right] . \underline{A} = \alpha_d + \underline{A}^H . \underline{\underline{R}}_o . \underline{A}$$

Which implies that the following expression is also valid for the beamformer

$$\underline{A} = \frac{\underline{\underline{R}}_0^{-1} . \underline{S}_d}{\underline{S}_d^H . \underline{\underline{R}}_0^{-1} . \underline{S}_d}$$

Phased array—Optimum SRB





CONSTRAINED BEAMFORMING

In addition to the DOA of the desired, it may be possible that other directions where a-priori it is known that interference or clutter impinge the aperture can be included on the constrains.

When NI directions of interference are known, the set of constrains will be:

$$\underline{A}^H . \left[\underline{S}_d \quad \underline{S}_{i,1} \quad . \quad \underline{S}_{i,NI} \right] = \left[1 \quad 0 \quad . \quad 0 \right]$$

-Note that NI+1 degrees of freedom are used thus only Q-NI-1 interferers can be attenuated on the adaptive mode of the beamformer.
-With this set of constrains, the beamformer in the quiescent mode will perfectly null out the NI interferers.
-It is crucial that the set of directions are precise, otherwise degrees of freedom are consumed without removing effectively the interferences.
-Since nuling an interference, in practice, represents 20 or 30 dB od attenuation, in order to do not increase the dynamic of the beamformer weights or to decrease sensitivity to missmatch, it is better to set the attenuation to some value (0.01 or 0.001) greater than cero

The design equations are:

$$\underline{A}^{H}.\left[\underline{S}_{d} \quad \underline{S}_{i,1} \quad . \quad \underline{S}_{i,NI}\right]=\left[1 \quad 0 \quad . \quad 0\right]$$

or

$$\underline{A}^{H}\underline{\underline{C}}=\underline{f}^{H}$$

$$\left.\underline{A}^{H}\underline{\underline{R}}\underline{A}\right|_{\min}$$

The solution to this problem is:

$$\underline{A}_{optimo}=\underline{\underline{R}}^{-1}.\underline{\underline{C}}.\left[\underline{\underline{C}}^{H}.\underline{\underline{R}}^{-1}.\underline{\underline{C}}\right]^{-1}.\underline{f}$$

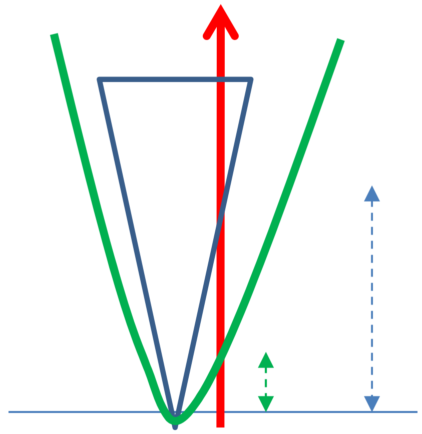
The quiescent response, i.e. the beamformer satisfying the constraints and with minimum response to the front-end noise (minimum norm) is:

$$\underline{\underline{C}}^{H}.\underline{A}=\underline{f}$$

$$\underline{A}^{H}.\sigma^{2}.\underline{\underline{I}}.\underline{A}=\sigma^{2}.\underline{A}^{H}.\underline{A}\propto\left\|\underline{A}\right\|^{2} \quad \Rightarrow \quad \underline{A}_{quiescent}=\underline{A}_{q}=\underline{\underline{C}}.\left[\underline{\underline{C}}^{H}.\underline{\underline{C}}\right]^{-1}.\underline{f}$$

Derivative Constrains

Let us imagine, that the direction of the interference is not known precisely. In this case, since the nulls use to be very deep, the resulting attenuation could be clearly not sufficient.



To alleviate this problem it is convenient to set derivative constraints on the beamformer response.

The original constrains set is:

$$\underline{A}^H.[\underline{S}_d \quad \underline{S}_i] = [1 \quad \beta]$$

with

$$\underline{S}_i(q) = \exp\left[j.2\pi f.d_q.\frac{sen(\theta_i)}{c}.\cos(\varphi_i - \varphi_q) \right]$$

Aiming to include derivative constrains, the derivative of the array factor with respect to the elevation at the interference location is:

$$\frac{\partial\left[\underline{A}^{H}.\underline{S}_{i}\right]}{\partial\theta_{i}}=\underline{A}^{H}.\frac{\partial\underline{S}_{i}}{\partial\theta_{i}}=\underline{A}^{H}.\underline{S}_{i}^{\theta}$$

being

$$\underline{S}_{i}^{\theta}(q)=\left(j.2\pi f.d_{q}.\frac{\cos(\theta_{i})}{c}.\cos\left(\varphi_{i}-\varphi_{q}\right)\right)\exp\left[j.2\pi f.d_{q}.\frac{sen(\theta_{i})}{c}.\cos\left(\varphi_{i}-\varphi_{q}\right)\right]$$

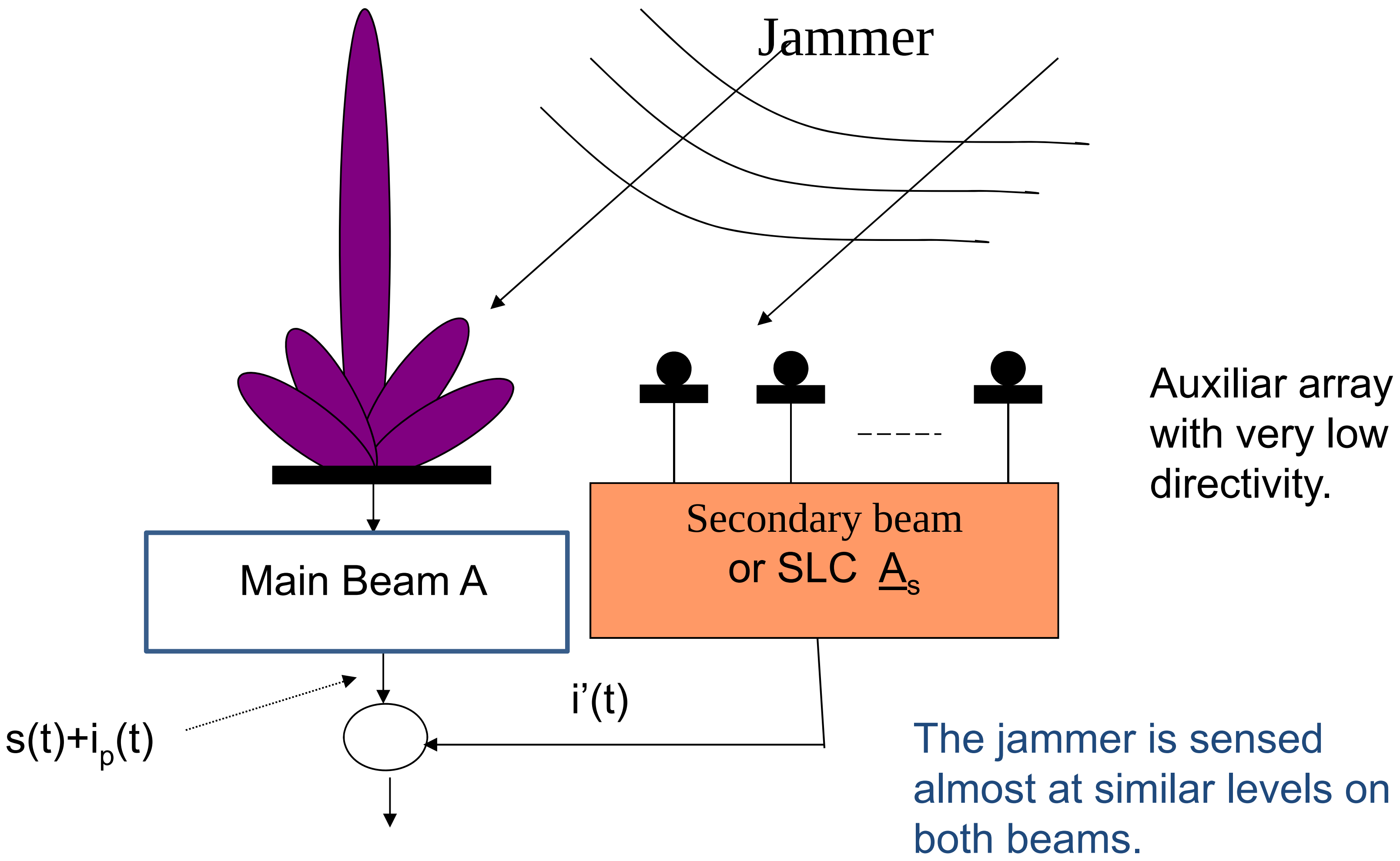
This can be used also for the azimuth

$$\underline{S}_{i}^{\theta}(q)=\left(-j.2\pi f.d_{q}.\frac{\cos(\theta_{i})}{c}.sen\left(\varphi_{i}-\varphi_{q}\right)\right)\exp\left[j.2\pi f.d_{q}.\frac{sen(\theta_{i})}{c}.\cos\left(\varphi_{i}-\varphi_{q}\right)\right]$$

Of course, derivative constrains can be used also in order to ensure proper reception of the desired when pointing errors occur. Then, the global set of constrains would be:

$$\underline{A}^{H}.\left[\underline{S}_{d}\quad\underline{S}_{d}^{\theta}\quad\underline{S}_{d}^{\varphi}\quad\underline{S}_{i}\quad\underline{S}_{i}^{\theta}\quad\underline{S}_{i}^{\varphi}\right]=\left[1\quad 0\quad 0\quad\beta\quad 0\quad 0\right]$$

The Sidelobe Canceller (SLC)



The error between the output of these two beams is:

$$\varepsilon(t) = s(t) + i_p(t) - \underline{A}_s^H . \underline{X}_{s,n}$$

The secondary beam is designed in such a way that minimizes the power of this error.

The snapshot on the secondary beam is:

$$\underline{X}_{s,n} = \alpha_d(t).\underline{S}_d + \beta.i_p(t).\underline{S}_i + \underline{w}_t \cong \beta.i_p(t).\underline{S}_i + \underline{w}_t$$

$$\underline{X}_{s,n} = i(t).\underline{S}_i + \underline{w}_s(t)$$

It is CRUCIAL that the target is not sensed in the secondary, otherwise it will be cancellation of it at the global output.

Thus, the secondary beamformer is:

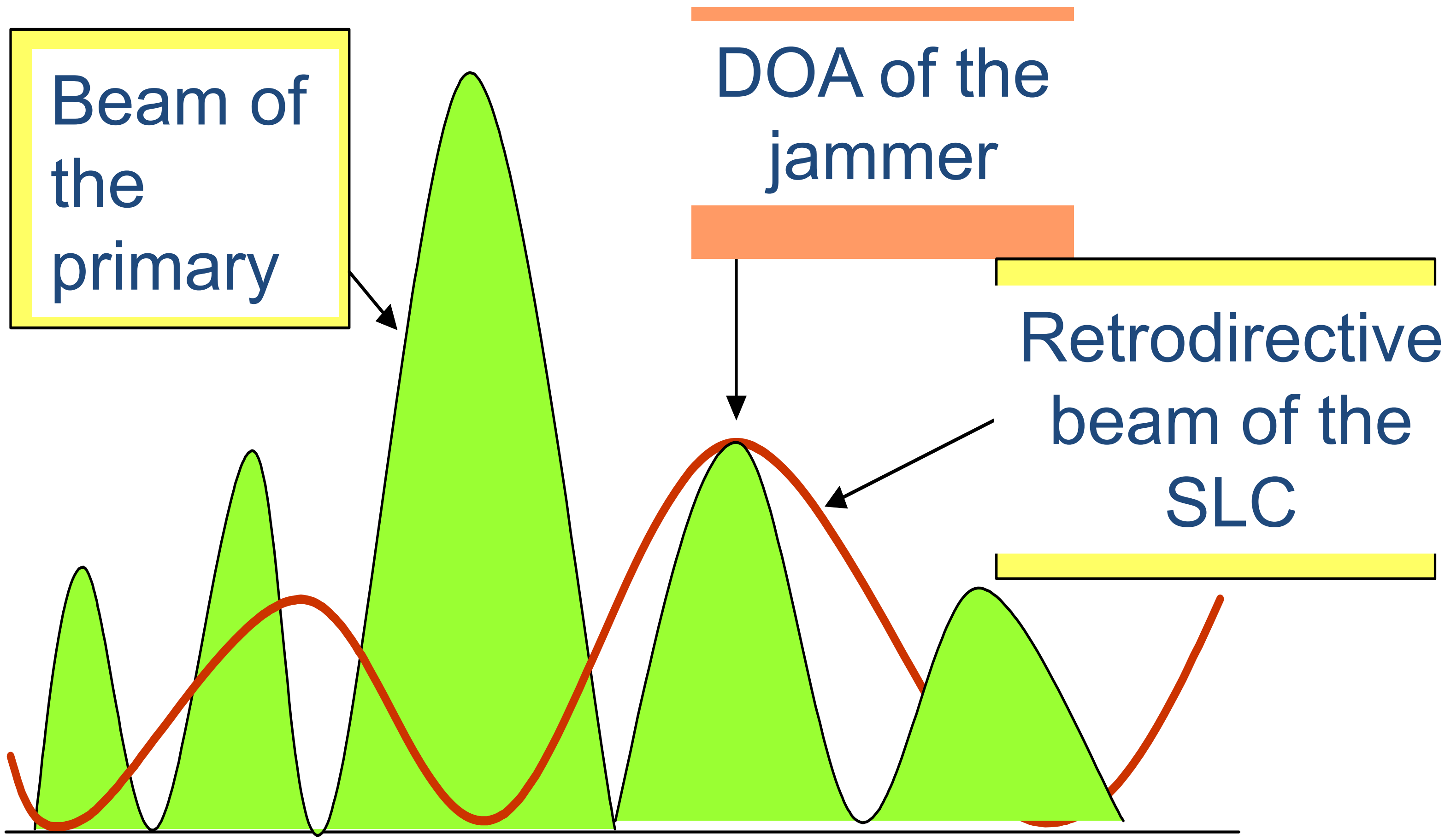
$$\underline{A}_s = \left[P_i.\underline{S}_i.\underline{S}_i^H + \sigma^2.\underline{\underline{I}}\right]^{-1}.\gamma.P_i.\underline{S}_i$$

$$\underline{A}_s = \frac{1}{\sigma^2}.\left[\underline{\underline{I}} - \frac{P_i.\underline{S}_i.\underline{S}_i^H}{\sigma^2 + Q.P_i}\right].\gamma.P_i.\underline{S}_i = \frac{\gamma.P_i}{\sigma^2 + Q.P_i}.\underline{S}_i$$

The global output is:

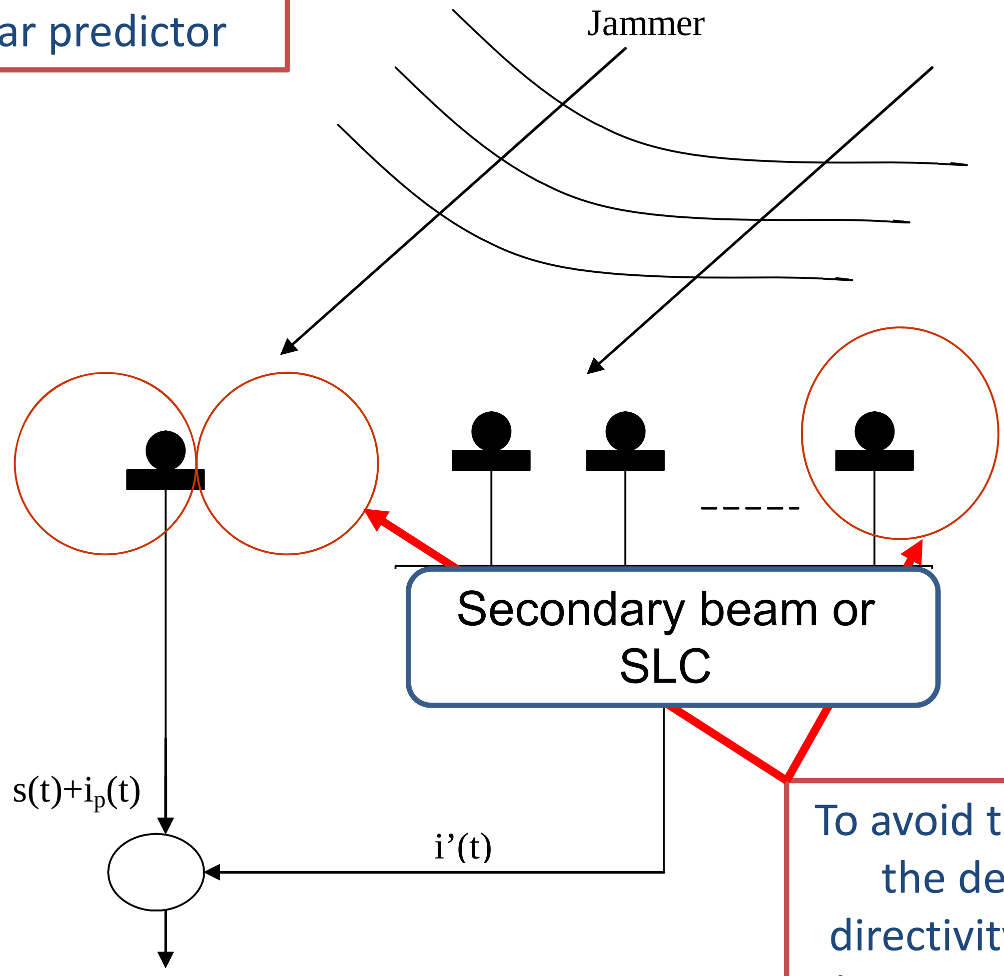
$$s(t)+\gamma.i(t)-\underline{A}_s^H.\underline{X}_{s,n}=s(t)+i(t).\left[\frac{1}{1+\frac{Q.P_i}{\sigma^2}}\right]+w_p(t)+\underline{A}_s^H.\underline{w}_s(t)$$

Note that the jammer is reduced proportionally to its power. In other words the SLC inverts the original SNR of the jammer that pass to be 1/SNR. This quality sometimes is used to refer the MSE as inverter of the interference SNR. Not also that the reduction is favored with the number of antennas in the secondary aperture.



The SLC forms a retrodirective beam such that the difference of the primary and the SLC produces a zero on the DOA of the jammer

Without desired, the SLC works as a linear predictor



To avoid the cancellation of the desired different directivity can be used for the primary and secondary (Example: Cardiod/Omnidirectional microphones)

THE GENERALIZED SLC or GSLC

Aiming to solve these two problems:
-Change only the degrees of freedom when the scenario changes.
-A new architecture were adaptive algorithms are easy to implement.

$$\underline{A}^{H}.\underline{\underline{C}} = \underline{f}^{T}$$

$$\left.\underline{A}^{H}.\underline{\underline{R}}.\underline{A}\right|_{min}$$

A new architecture to implement the constrained beamformer is going to be presented based on its design equations

$$\underline{A}_{optimo} = \underline{\underline{R}}^{-1}.\underline{\underline{C}}.\left[\underline{\underline{C}}^{H}.\underline{\underline{R}}^{-1}.\underline{\underline{C}}\right]^{-1}.\underline{f}$$

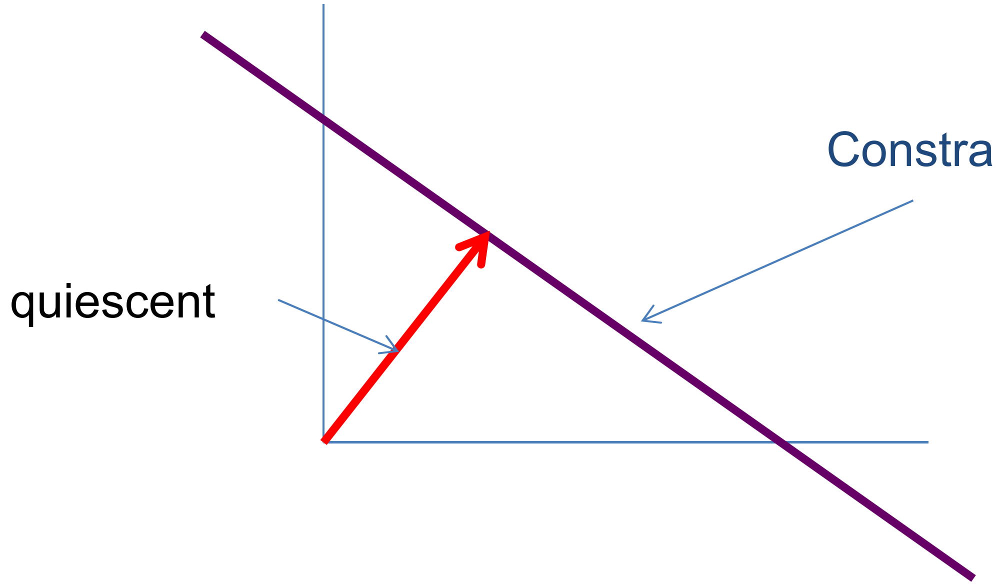
When there is not sources in the scenario, the so-called quiescent solution is:

$$\underline{A}_{q} = \underline{\underline{C}}.\left[\underline{\underline{C}}^{H}.\underline{\underline{C}}\right]^{-1}.\underline{f}$$

Then, it can be assumed that when sources impinge the aperture the optimum beamformer will be the combination of the quiescent with an additional, so-called adaptive beamformer

$$\underline{A} = \underline{A}_{q} + \underline{A}_{l}$$

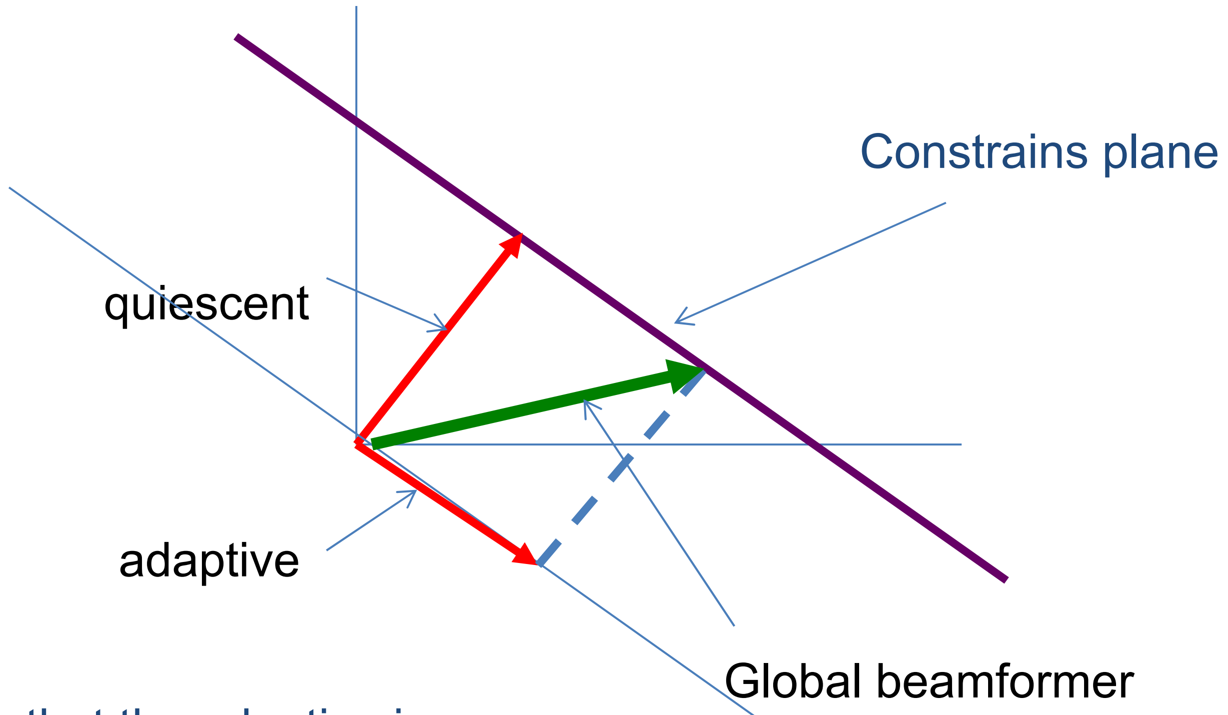
Since the quiescent is the vector that, satisfying the constrains is of minimum norm, a geometric interpretation (for the case of Q=2 antennas) will be:



Note that the quiescent is orthogonal to the constrains since its projection on the plane is zero

$$\left(\underline{\underline{I}} - \underline{\underline{C}}.\left(\underline{\underline{C}}^H.\underline{\underline{C}}\right)^{-1}\underline{\underline{C}}^H\right).\underline{A}_q = \underline{\underline{P}}_C.\underline{A}_q = \underline{0}$$

The adaptive beamformer has to be orthogonal to the quiescent, since this ensures that the addition of both stay on the contrains plane.



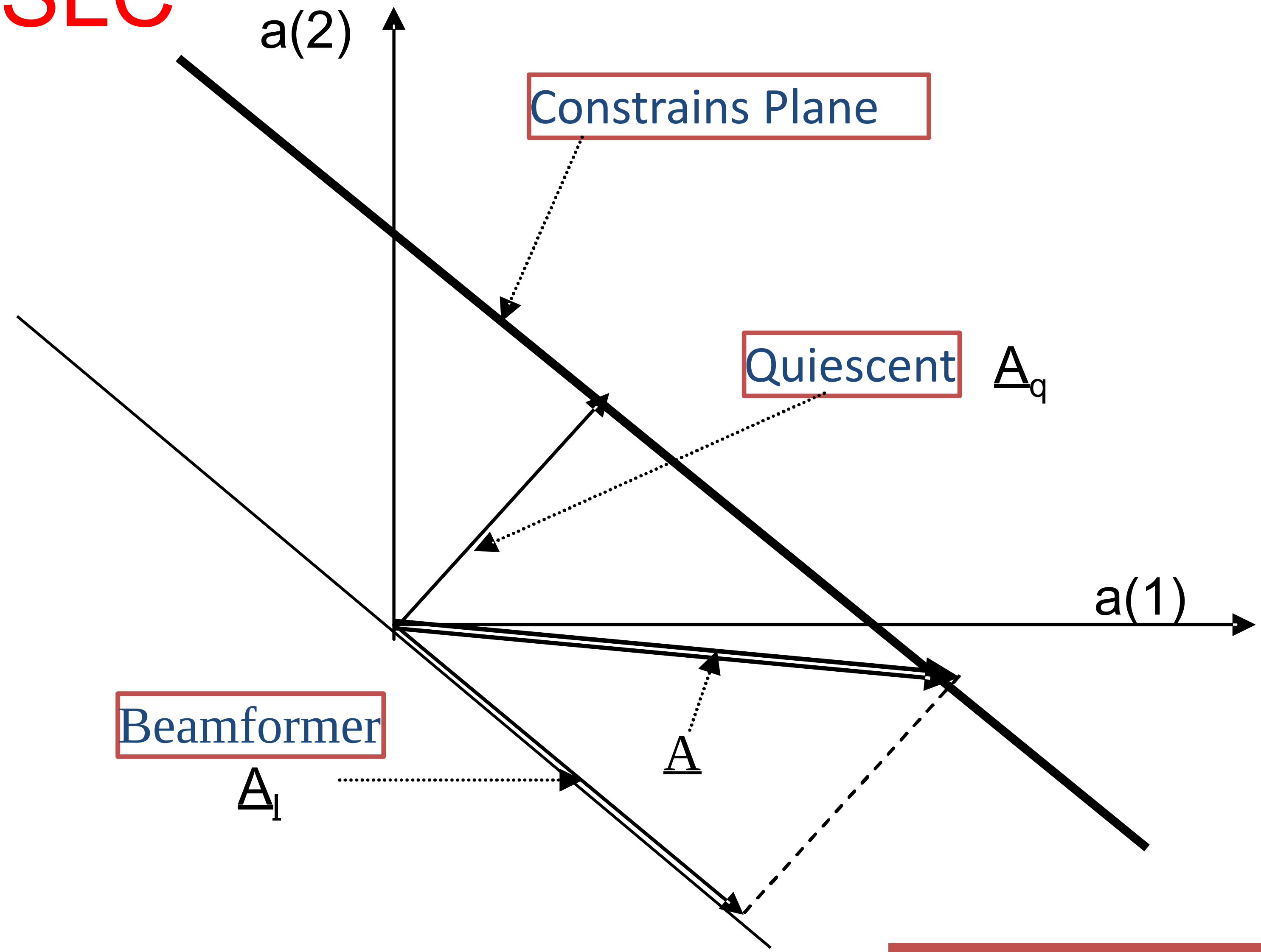
To ensure that the adaptive is orthogonal to the quiescent a matrix B is needed.

$$\underline{A}_l = \left[\left(\underline{\underline{I}} - \underline{\underline{C}}.\left(\underline{\underline{C}}^H.\underline{\underline{C}}\right)^{-1}\underline{\underline{C}}^H\right)\right].\underline{A}_o \triangleright \underline{\underline{B}}^H.\underline{A}_o$$

$$\underline{\underline{B}}^H.\underline{\underline{C}} = \underline{\underline{0}}$$

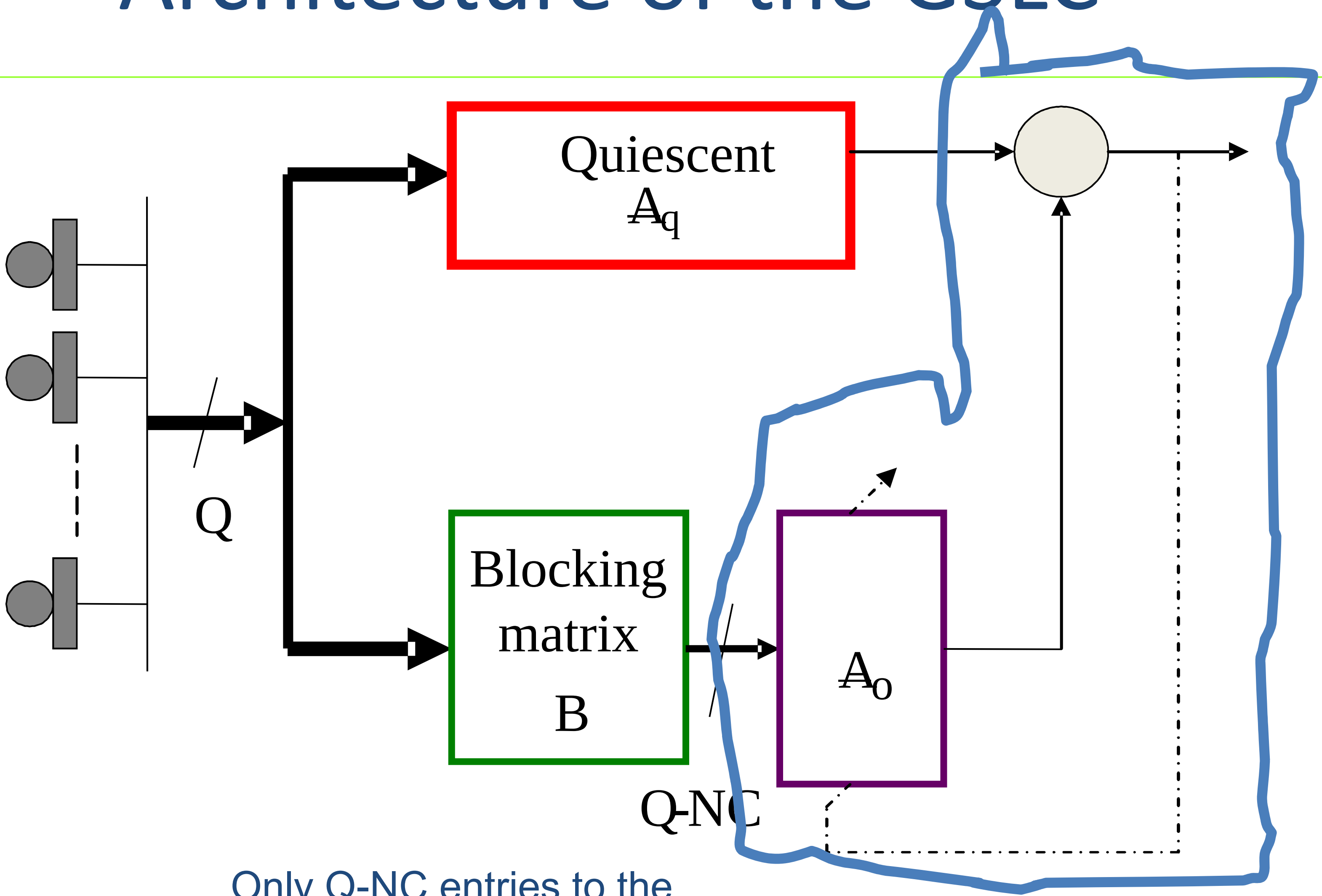
This is the so-called BLOCKING matrix of the GSLC

GSLC



Geom.Interpretation

Architecture of the GSLC



Only Q-NC entries to the adaptive module

This is a Wiener filter

The design of the Blocking matrix

The blocking matrix has to be orthogonal to the constrain matrix, but, at the same time has to be an orthogonal matrix itself in order to do not introduce colored noise to the adaptive beamformer.

$$\underline{\underline{B}}^H . \underline{\underline{C}} = \underline{\underline{0}}$$

$$\underline{\underline{B}}^H \underline{\underline{B}} = \underline{\underline{I}}$$

The solution to this problem is easy since the constrains matrix is rank deficient, i.e. its dimensions are QxNC with NC<Q. Thus, using its svd, it will be Q-NC null eigenvectors that can be used to form the blocking matrix.

$$\underline{\underline{C}} = \underline{\underline{U}}.\underline{\underline{D}}.\underline{\underline{V}}^H = \begin{bmatrix} \underline{u}_1 & . & \underline{u}_{NC} & . & \underline{u}_Q \end{bmatrix} \begin{bmatrix} \psi_1 & . & 0 & 0 & . & 0 \\ . & . & 0 & 0 & . & 0 \\ 0 & . & \psi_{NC} & 0 & . & 0 \end{bmatrix} \begin{bmatrix} \underline{v}_1^H \\ . \\ \underline{v}_{NC}^H \end{bmatrix}$$

The optimum blocking matrix is:

$$\underline{\underline{B}} = \begin{bmatrix} \underline{u}_{NC+1} & . & \underline{u}_Q \end{bmatrix}$$

When the coloring of the noise constrain is removed (tolerated for the global performance of the GSLC), there is the possibility of using blocking matrixes with low number of entries different from zero (low complexity)

Example:

Since the DOA of the desired is always the first constrain, we start with it. To block this direction we may use the following matrix:

$$\underline{\underline{C}} \rightarrow \underline{S}_d$$

$$\underline{A}_q = \frac{\underline{S}_d}{Q} \propto \underline{S}_d$$

$$\underline{\underline{B}}_d = \begin{bmatrix} 1 & \exp\left(j(\phi_1 - \phi_2)\right) & 0 & . & 0 \\ 0 & 1 & \exp\left(j(\phi_2 - \phi_3)\right) & . & 0 \\ 0 & 0 & 1 & . & 0 \\ . & . & . & . & . \\ 0 & 0 & . & 1 & \exp\left(j(\phi_{Q-1} - \phi_Q)\right) \end{bmatrix}$$

where

$$\phi(1) = \frac{2\pi.f}{c} d_1 \sin(\theta_d)\cos(\varphi_1 - \varphi_d)$$

$$\phi(2) = \frac{2\pi.f}{c} d_2 \sin(\theta_d)\cos(\varphi_2 - \varphi_d)$$

Now, let us assume that there is another constrain (interference), the new C matrix will be:

$$\underline{\underline{C}} \rightarrow \left[\underline{S}_d \; \underline{S}_i\right]$$

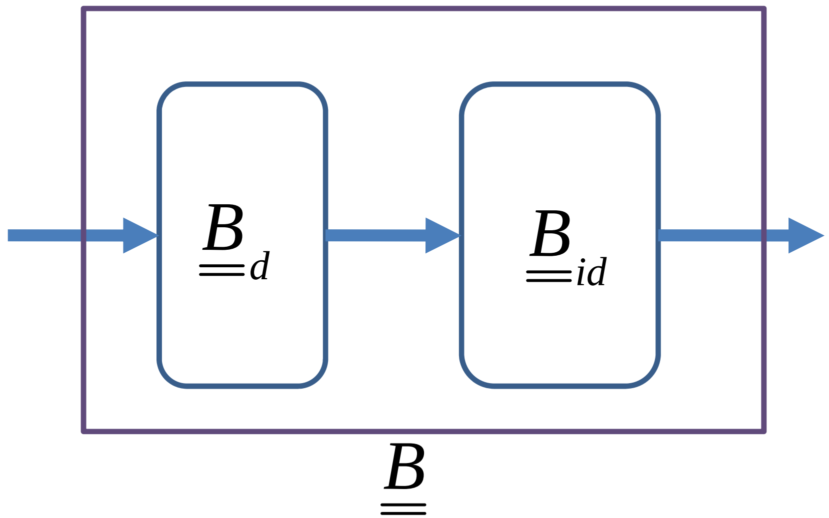
Passing the new DOA for the already designed blocking for the desired, we have:

$$\underline{\underline{B}}_d^H \, \underline{S}_i = \underline{S}_{id}$$

The second blocking matrix is designed as before, just blocking the Sid vector.

$$\underline{\underline{B}}_{id}^H \, \underline{S}_{id} = \underline{0}$$

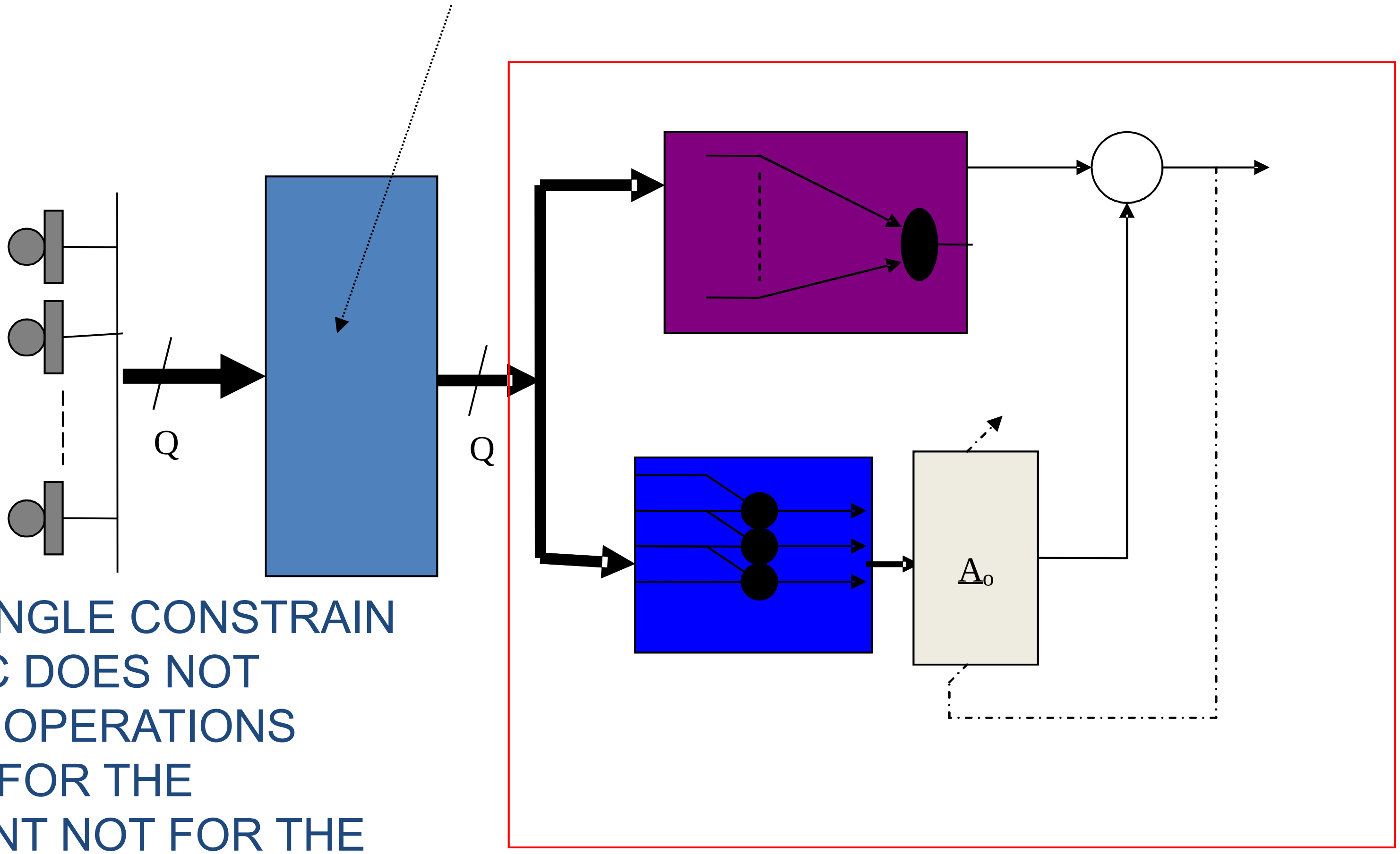
The blocking matrix is formed by the cascade of the two block designed



NOTE that the number of outputs for every direction from the constrains, decreases one reflecting theuseof a degree of freedom

GSLC with pre-steering

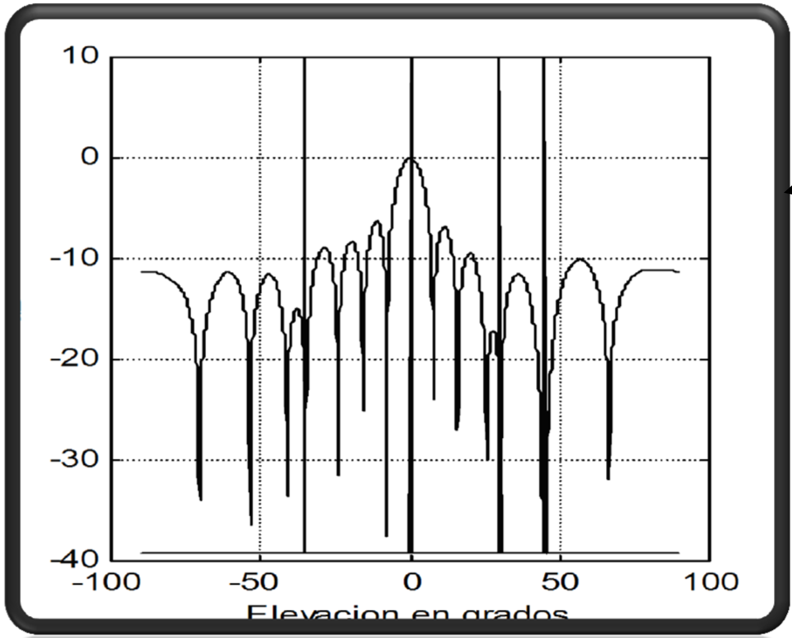
$$\underline{\underline{G}}(\theta_d, \varphi_d) = diagonal\left\lfloor -\exp\left(j\phi_q\right); q = 1, Q \right\rfloor$$

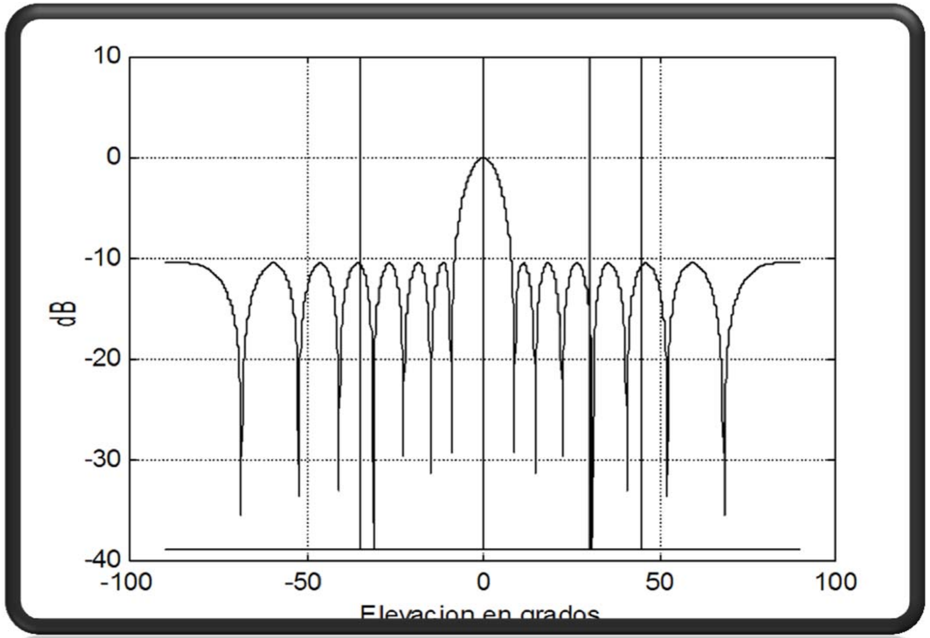


WITH A SINGLE CONSTRAIN THE GSLC DOES NOT REQUIRE OPERATIONS NEITHER FOR THE QUIESCENT NOT FOR THE BLOCKING MATRUX

QUIESCENT Control

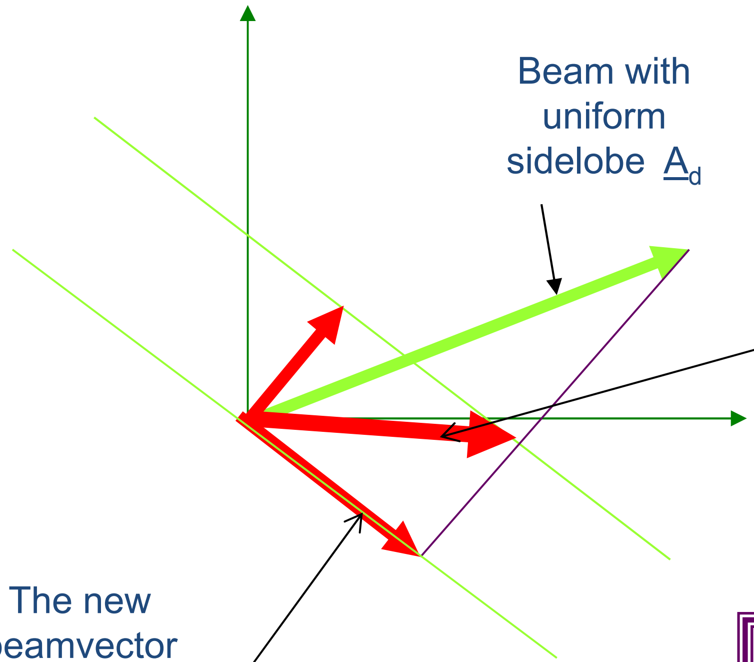
GSLC matched with phased array quiescent





GSLC quiescent with uniform sidelobe quiescent

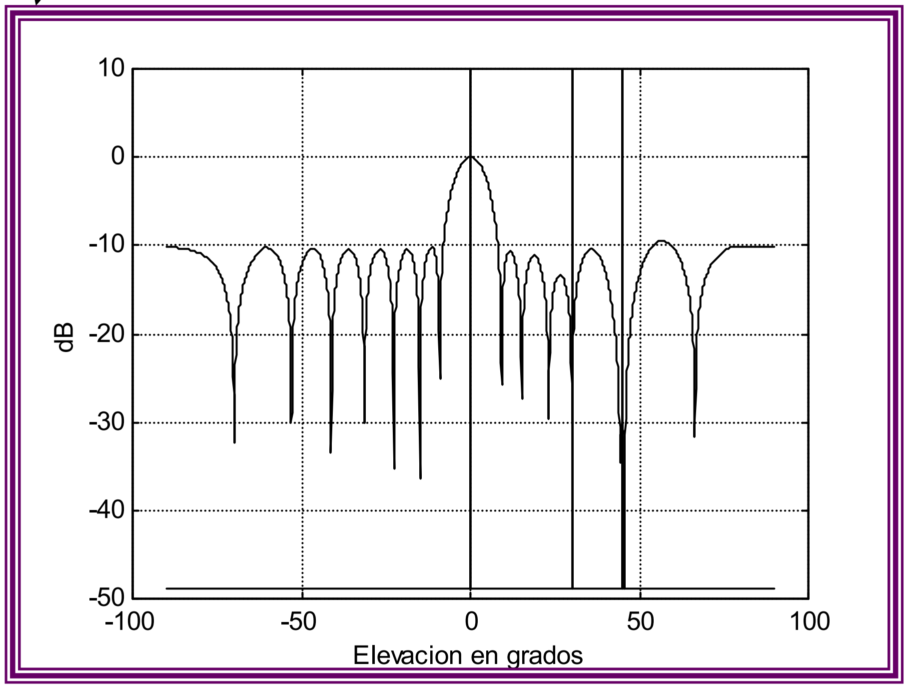
Since the desired quiescent may not hold the constrains, the closest vector has to be used in the constrain plane



Beam with uniform sidelobe $\underline{A}_d$

The closest vector to the desired quiescent holding the constrains $\underline{A}_{qc}$

The new beamvector has a component different from zero at the blocking matrix output $\underline{A}_{dq}$



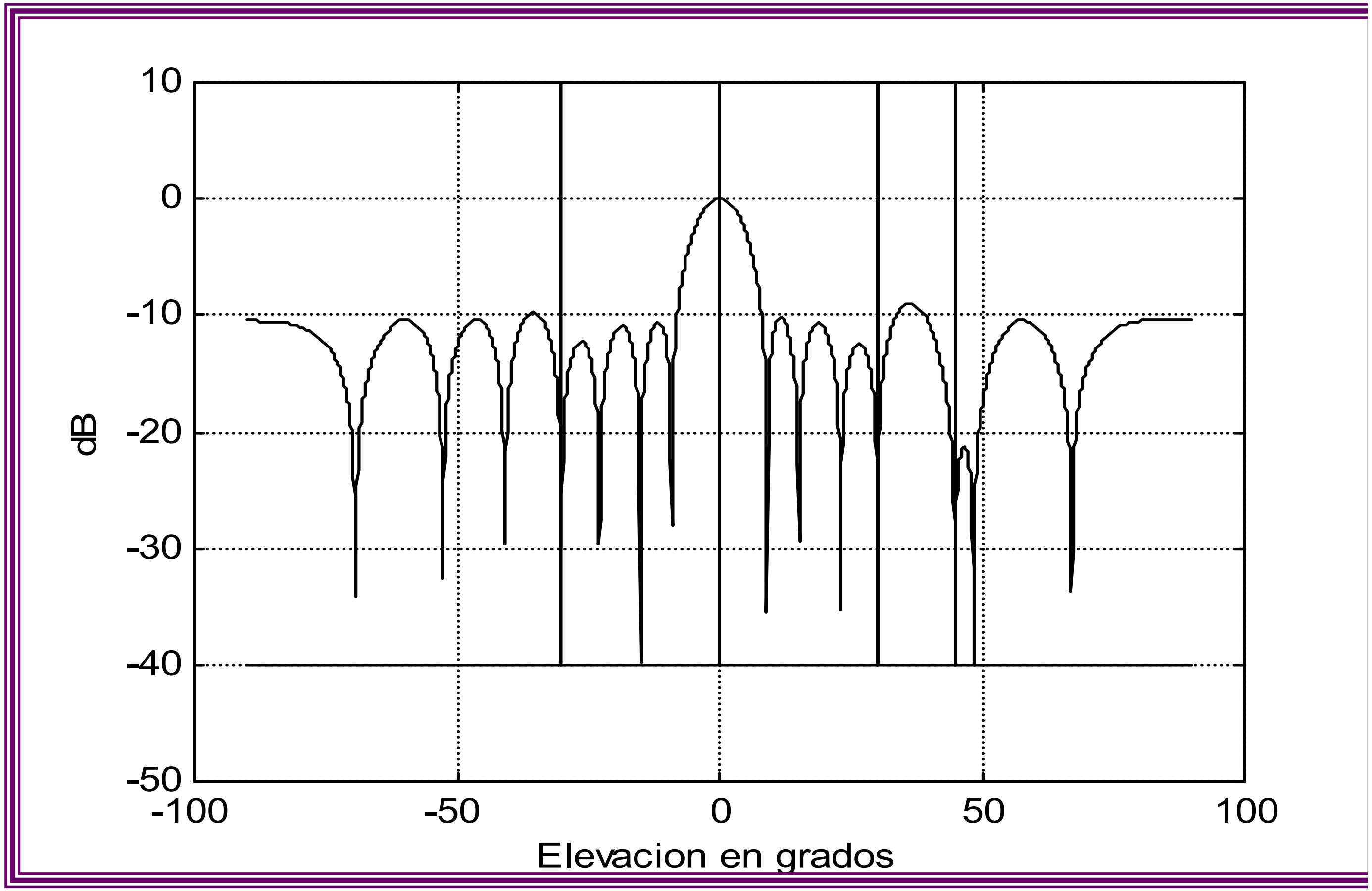
GSLC with quiescent control

New “shape” constrain

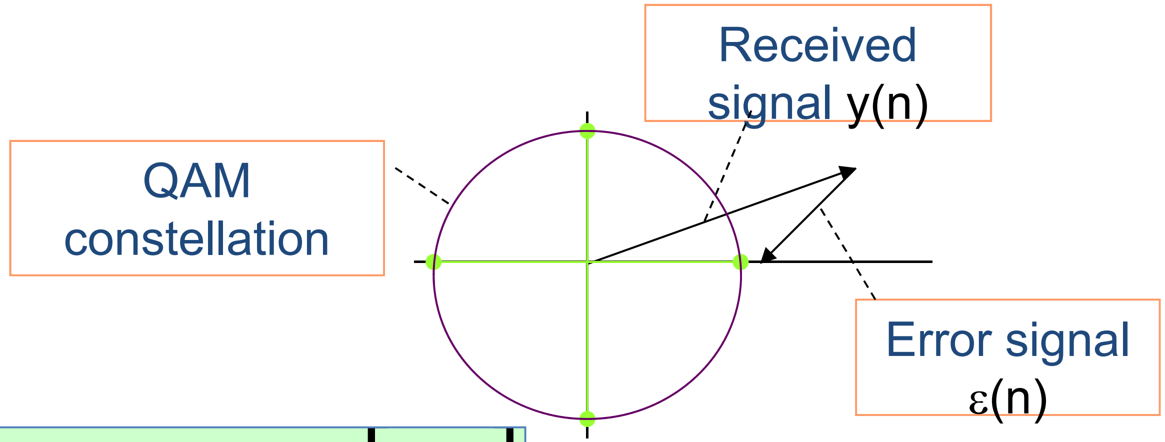
$$\underline{\underline{C}}_N^H = \begin{bmatrix} \underline{\underline{C}}^H \\ \underline{A}_{dq}^H \end{bmatrix} \quad y \quad \underline{f}_N = \begin{bmatrix} f \\ \delta \end{bmatrix}$$

where

$$\delta = \underline{A}_{dq}^H \cdot \underline{A}_{q,c}$$



ALPHABET REFERENCE



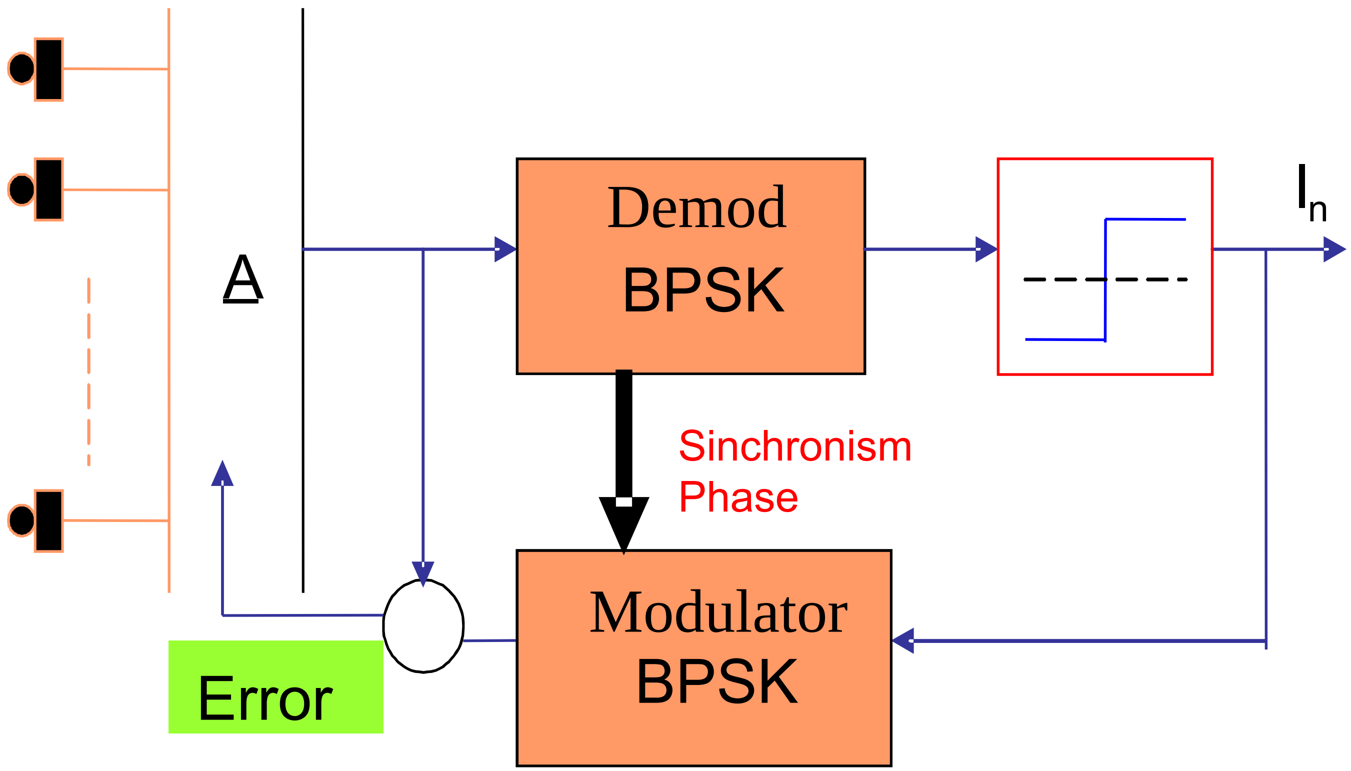
$$\underline{d}_n^T = NLC\left[\underline{y}_n^T\right]$$

Reference

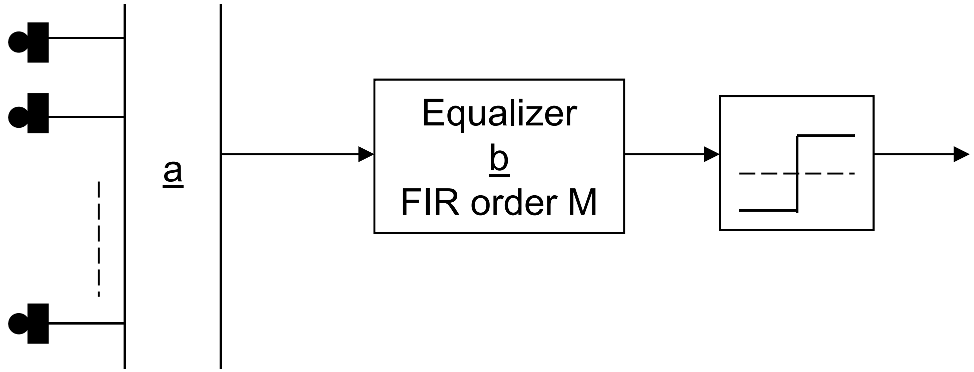
Beamformer

$$\underline{A}_{i+1} = \left[\underline{\underline{X}}_n . \underline{\underline{X}}_n^H\right]^{-1} . \underline{\underline{X}}_n . \underline{d}_n^*$$

The BPSK example



Beamformer plus forward equalizer

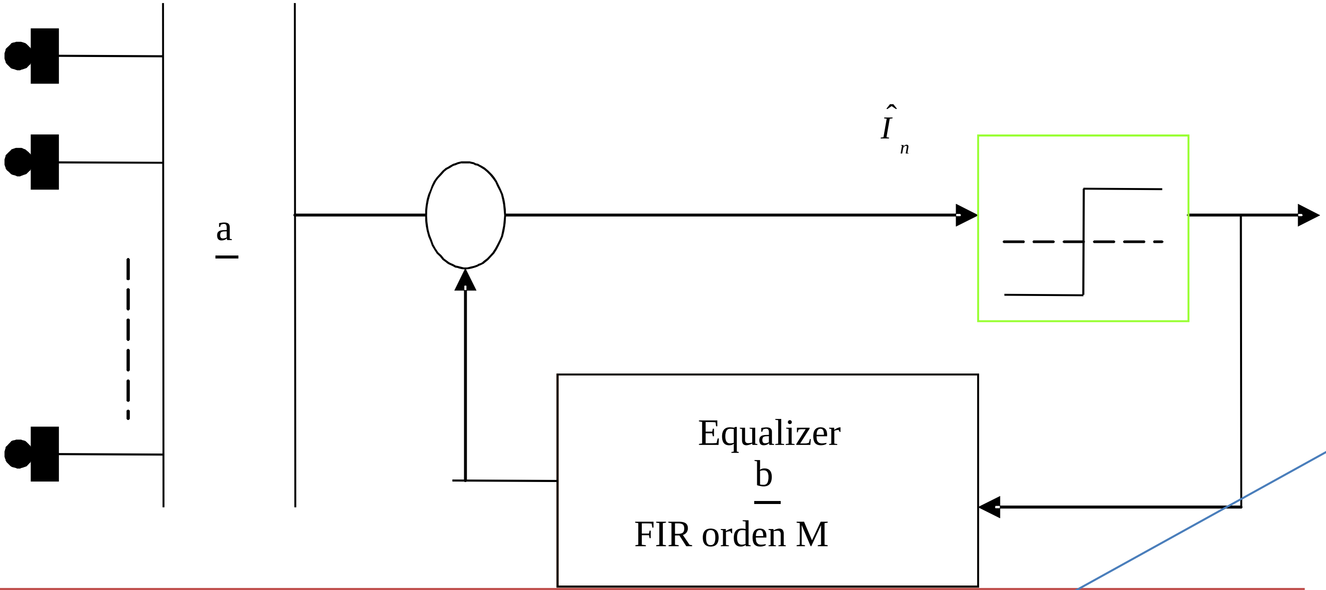


$$\hat{I}_n = \sum_{m=0}^{M} b(q).\underline{a}^H.\underline{X}_n = \underline{a}^H.\left[\underline{X}_n, \underline{X}_{n-1}, \ldots, \underline{X}_{n-M}\right]\underline{b} = \underline{a}^H.\underline{\underline{X}}_n.\underline{b}$$

$$\hat{I}_n = \underline{a}^H.\underline{\underline{X}}_n.\underline{b} = \underline{\alpha}^H.\underline{X}_{ne} = trace\left[\underline{\underline{A}}.\underline{\underline{X}}_n\right]$$

$$\underline{\underline{A}} = \lambda_{max}.\underline{b}.\underline{a}^H + \sum_{l=1}^{min(Q,M+1)} \lambda_l.\underline{u}_l.\underline{v}_l^H$$

Beamformer plus DFE



constrain

$$\underline{b}^H . \underline{1} = 1$$

$$\varepsilon(n) = I_n - \hat{I}_n = I_n - \underline{a}^H . \underline{X}_n + \sum_{p=1}^{P} b(p) I_{n-p} = \underline{b}^H . \underline{I}_n - \underline{a}^H . \underline{X}_n$$

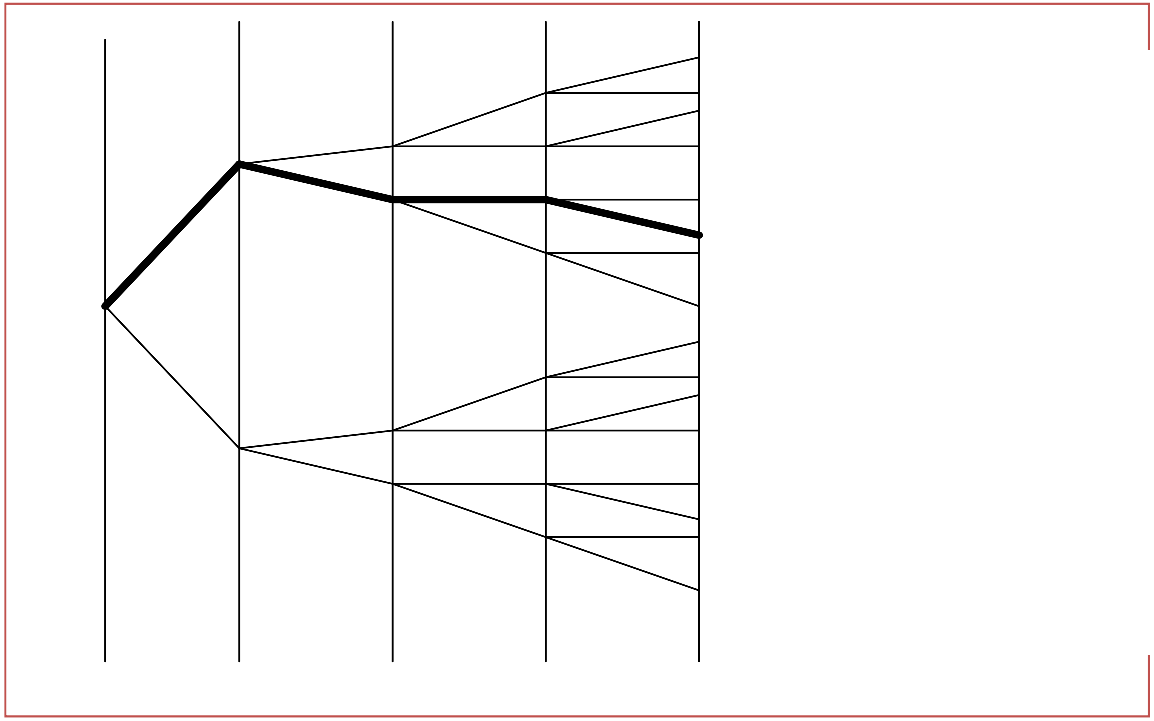
Where matrix G is the cross correlation between vector of reference and the received snapshot

$$\lambda = 1 - \underline{1}^H . \underline{\underline{G}} . \underline{a}$$

$$\underline{b} = \underline{\underline{G}}^H . \underline{a} - \lambda . \underline{1} = \left(\underline{\underline{I}} - \underline{1}\underline{1}^H\right) \underline{\underline{G}}^H \underline{a} + \underline{1}$$

$$\underline{\underline{R}}\underline{a} = \underline{\underline{G}}\underline{b} \Rightarrow \underline{a} = \left(\underline{\underline{R}} - \underline{\underline{G}}\left(\underline{\underline{I}} - \underline{1}\underline{1}^H\right)\underline{\underline{G}}^H\right)^{-1} . \underline{\underline{G}} . \underline{1}$$

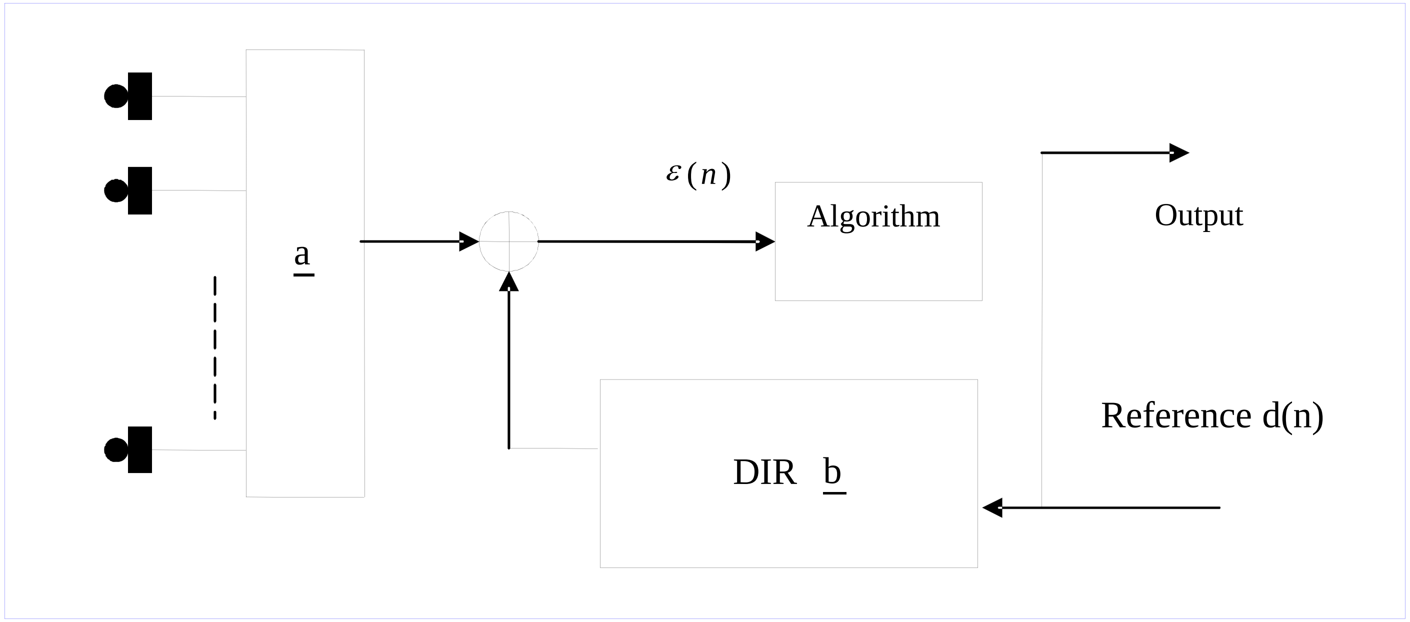
Sequence Detectors



Each vertical line denotes a symbol interval. From each node two branches for “1” and “0”. Each branch is labeled with the Euclidean distance D(.). Bold face the path corresponding to the sequence 1010.

$$D(I_n) = \left| \underline{b}^H . \underline{I}_n - \underline{a}^H . \underline{X}_n \right|^2 \quad ; \quad \forall I_n \in \mathrm{A}$$

The MDIR Receiver



The receiver snapshot and its covariance are:

$$\underline{X}_n = \underline{\underline{G}}.\underline{d}_n + \underline{X}_{w,n} \quad y \quad \underline{\underline{R}} = \underline{\underline{G}}.\underline{\underline{D}}.\underline{\underline{G}}^H + \underline{\underline{R}}_w$$

The generalized SNR at the array output can be defined as the response to the desired divided by the MSE error

$$SNR = \frac{\underline{a}^H.\underline{\underline{G}}.\underline{\underline{G}}^H.\underline{a}}{\xi}$$

The design will be to minimize the MSE, yet preserving, as in an AGC Automatic Gain Control, the power of the desired at the array output. In summary the design will be:

$$\xi = \underline{a}^H \underline{\underline{R}}\underline{a} + \underline{b}^H \underline{b} - \underline{a}^H \underline{\underline{G}}\underline{b} - \underline{b}^H \underline{\underline{G}}^H \underline{a}\Big|_{\min}$$

$$s.t. \qquad \underline{a}^H \underline{\underline{G}}\underline{\underline{G}}^H \underline{a} = kte.$$

The solution is, after deriving the corresponding Lagrangian:

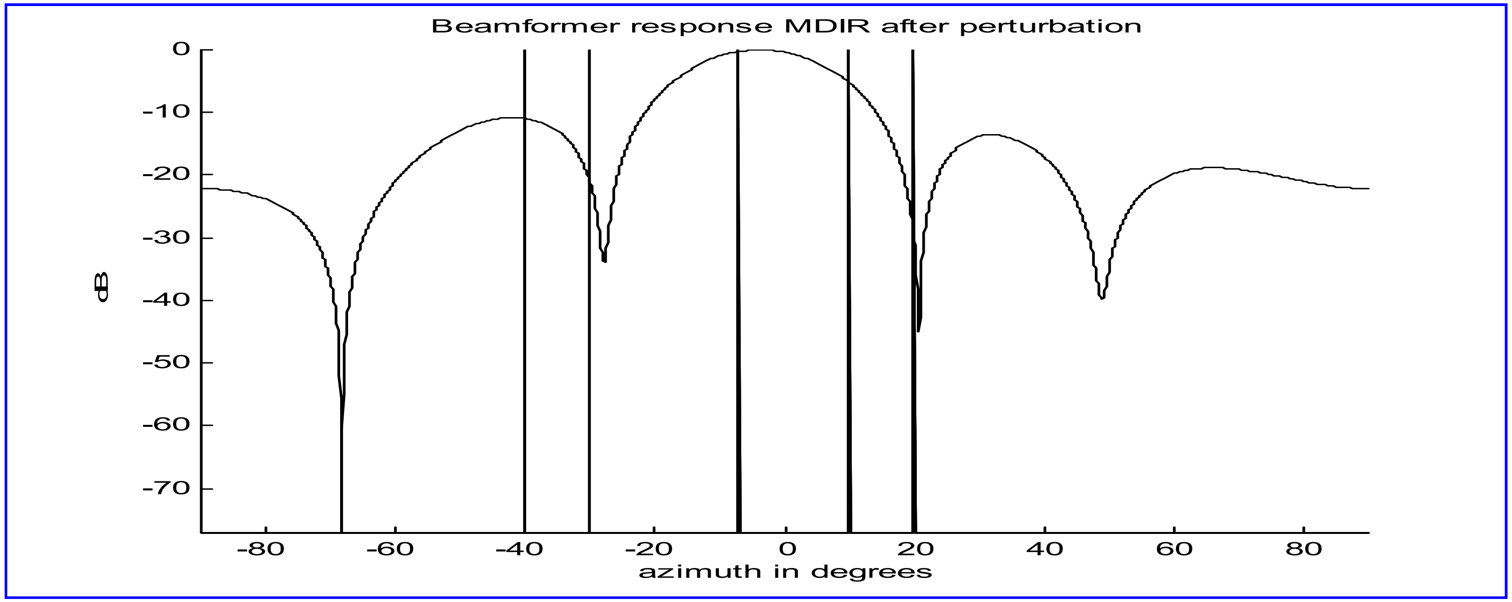
$$\underline{\underline{R}}.\underline{a} - \underline{\underline{G}}\underline{b} = \lambda.\underline{\underline{G}}.\underline{\underline{G}}^H.\underline{a}$$

$$\underline{b} = \underline{\underline{G}}^H.\underline{a}$$

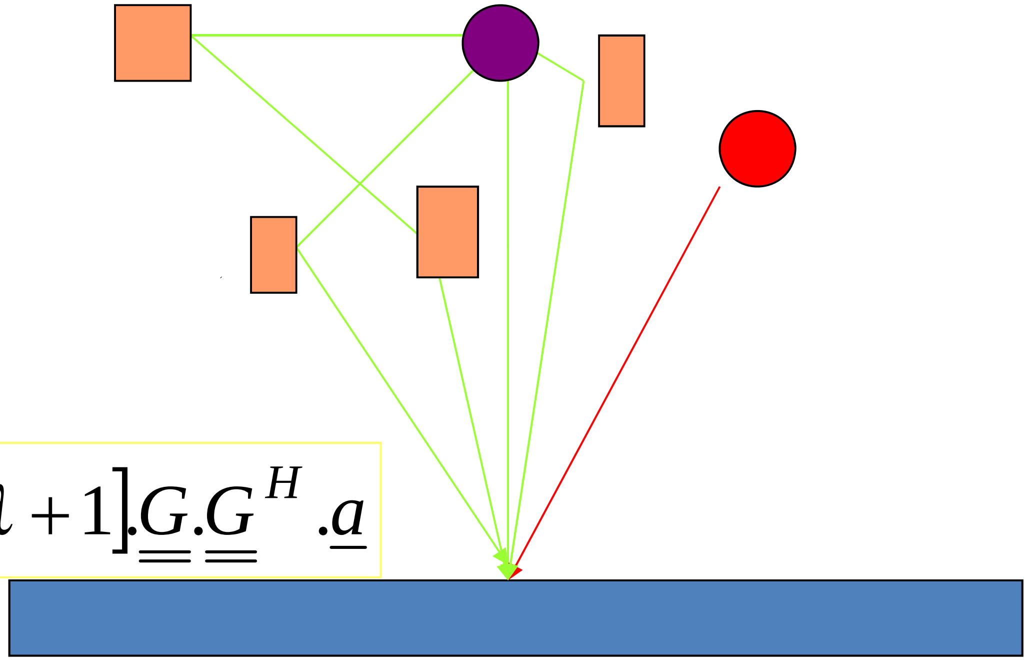
Where the beamformer is the eigenvector associated with the minimum eigenvalue and the DIR is MATCHED to the array response

$$\underline{\underline{R}}.\underline{a} - \underline{\underline{G}}\underline{b} = \lambda.\underline{\underline{G}}.\underline{\underline{G}}^H.\underline{a}$$

$$\underline{b} = \underline{\underline{G}}^H.\underline{a}$$

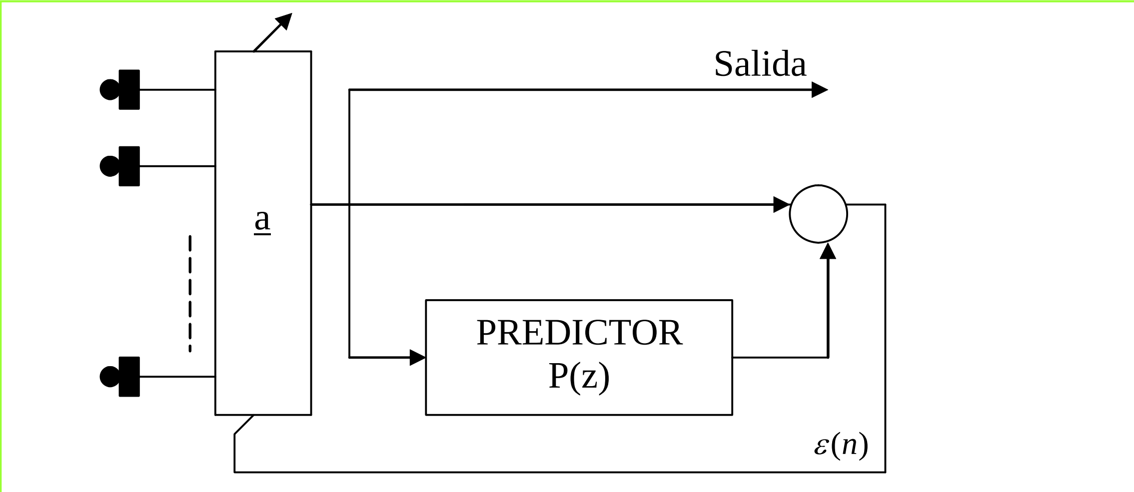


Escenario



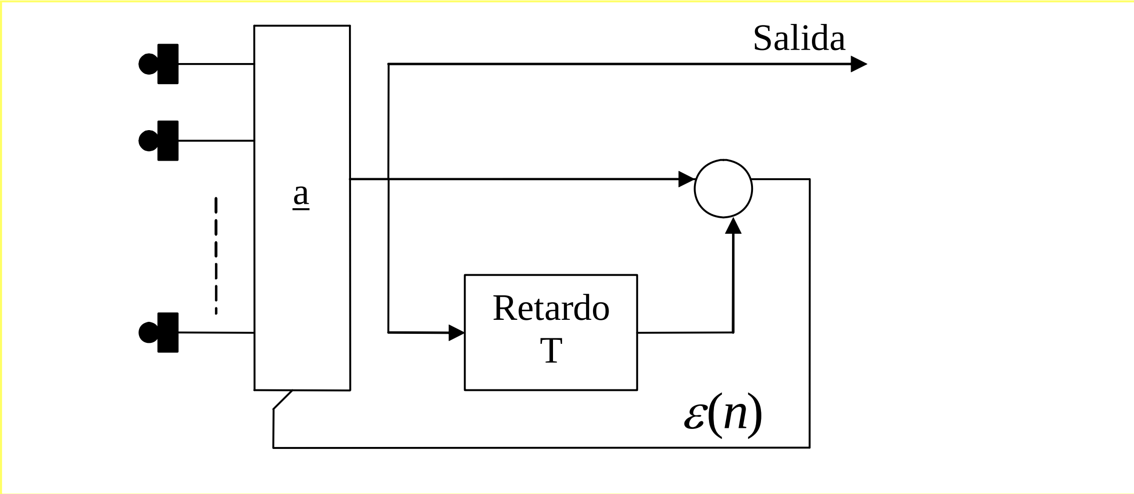
$$\underline{\underline{R}}.\underline{a} = \lambda_{min}.\underline{\underline{G}}.\underline{\underline{G}}^{H}.\underline{a} = [\lambda + 1].\underline{\underline{G}}.\underline{\underline{G}}^{H}.\underline{a}$$

Beamformer as Linear Predictor



General Architecture

For a single carrier the exact predictor is simple

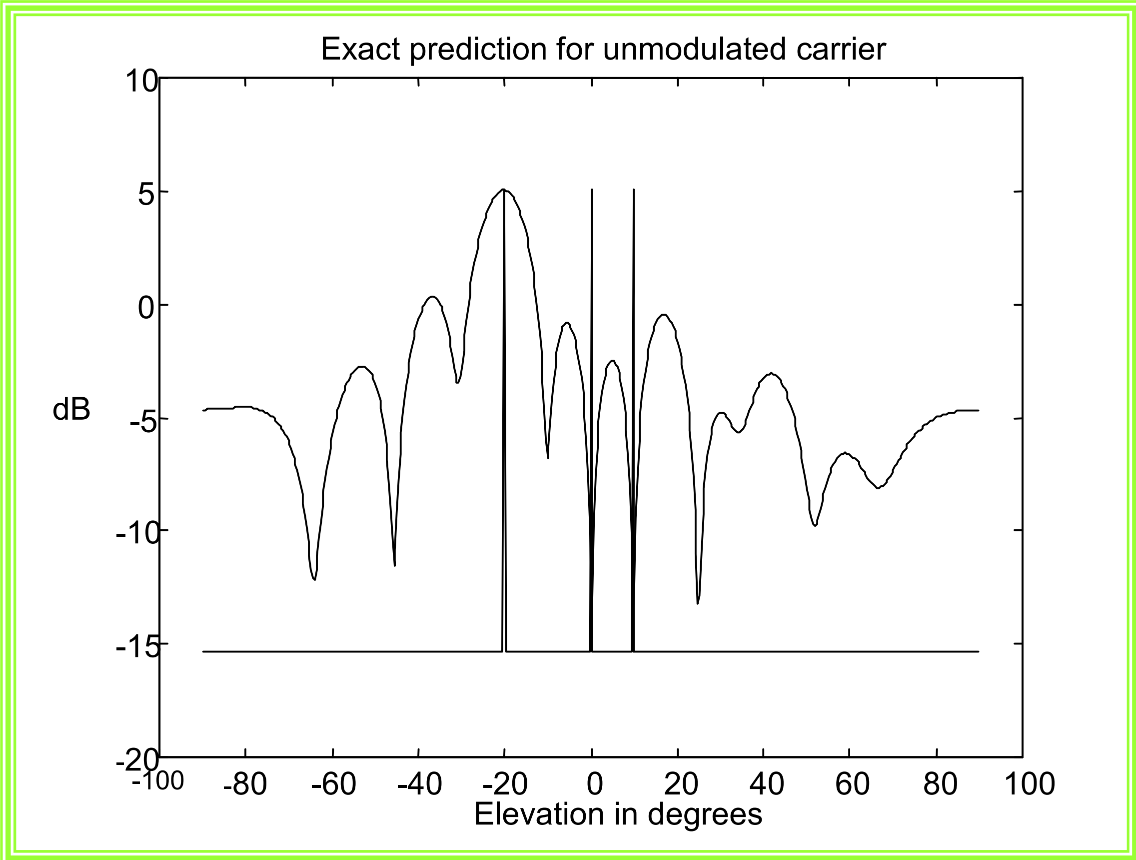


$$E\left[\left|\underline{a}^H.\left[\underline{X}_n - \underline{X}_{n-m}\right]\right|^2\right]_{\min} = \underline{a}^H.\left\{2.\underline{\underline{R}} - \left(\underline{\underline{R}}_m + \underline{\underline{R}}_m^H\right)\right\}.\underline{a} = \underline{a}^H.\left\{2.\underline{\underline{R}} - 2.\underline{\underline{R}}_c\right\}.\underline{a}$$

$$\underline{\underline{R}}_c = \frac{\underline{\underline{R}}_m + \underline{\underline{R}}_m^H}{2} \quad ; \quad \underline{\underline{R}}_m = E\left[\underline{X}_n.\underline{X}_{n-m}^H\right]$$

$$\underline{a}^H.\underline{\underline{R}}_c.\underline{a} = \phi_o$$

Constrain as AGC to avoid the trivial solution



Beamforming using Cyclostationarity

The acf of a cyclostationary process is:

$$r(t,\tau) = E\left[s(t+\tau/2).s^*(t-\tau/2)\right] = \sum_{m=-\infty}^{\infty} \beta_m(\tau).\exp(-jm\alpha.t)$$

The property that can be used as reference is:

$$E\left[s(t+\tau/2).s^*(t-\tau/2).\exp(j\alpha t)\right] = \beta_1(\tau) \neq 0$$

Assuming the cyclic frequency known (usually the baud rate), we collect the following snapshots from the aperture

$$\underline{X}_n \quad y \quad \underline{X}_{c,n} = \underline{X}_n.\exp(j\alpha t)$$

The objective is to design both beamformers (vectors a and c) in order to minimice the error……

$$\varepsilon(n) = \underline{a}^H . \underline{X}_n - \underline{c}^H . \underline{X}_{c,n}$$

The constrain will be that the cross-correlation among the output of both beamformers has to be different of zero when the desired signal, with a given cyclic frequency, is present.

$$\xi = \underline{a}^H . \underline{\underline{R}} . \underline{a} + \underline{c}^H . \underline{\underline{R}} . \underline{c} - \underline{a}^H . \underline{\underline{R}}_c . \underline{c} - \underline{c}^H . \underline{\underline{R}}_c^H . \underline{a} \Big|_{min}$$

$$\underline{a}^H . \underline{\underline{R}}_c . \underline{c} + \underline{c}^H . \underline{\underline{R}}_c^H . \underline{a} = \phi_o$$

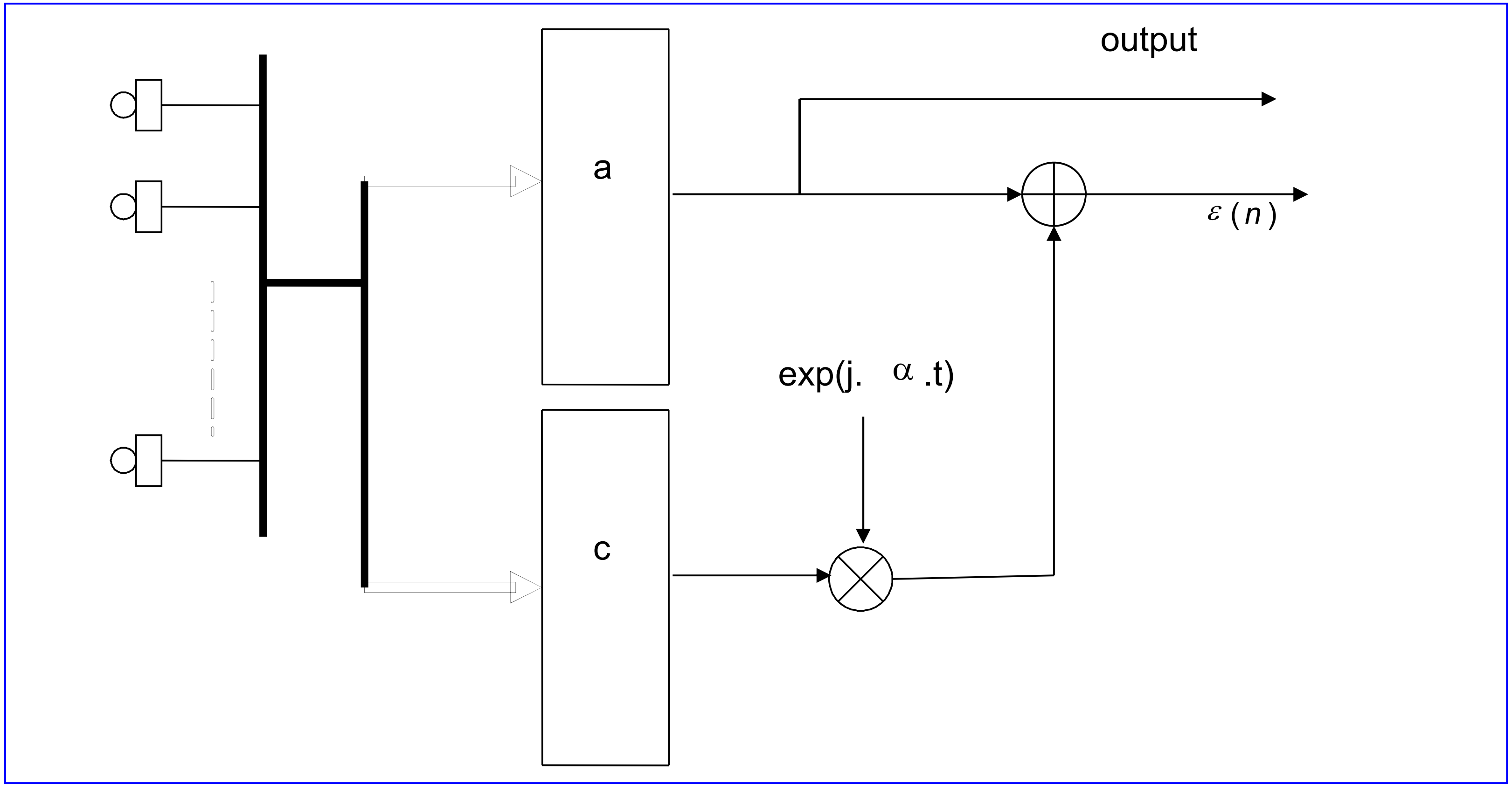
The solution is:

$$\underline{\underline{R}}\underline{a} = (\lambda + 1) \underline{\underline{R}}_c \underline{c}$$

$$\underline{\underline{R}}\underline{c} = (\lambda + 1) \underline{\underline{R}}_c^H \underline{a}$$

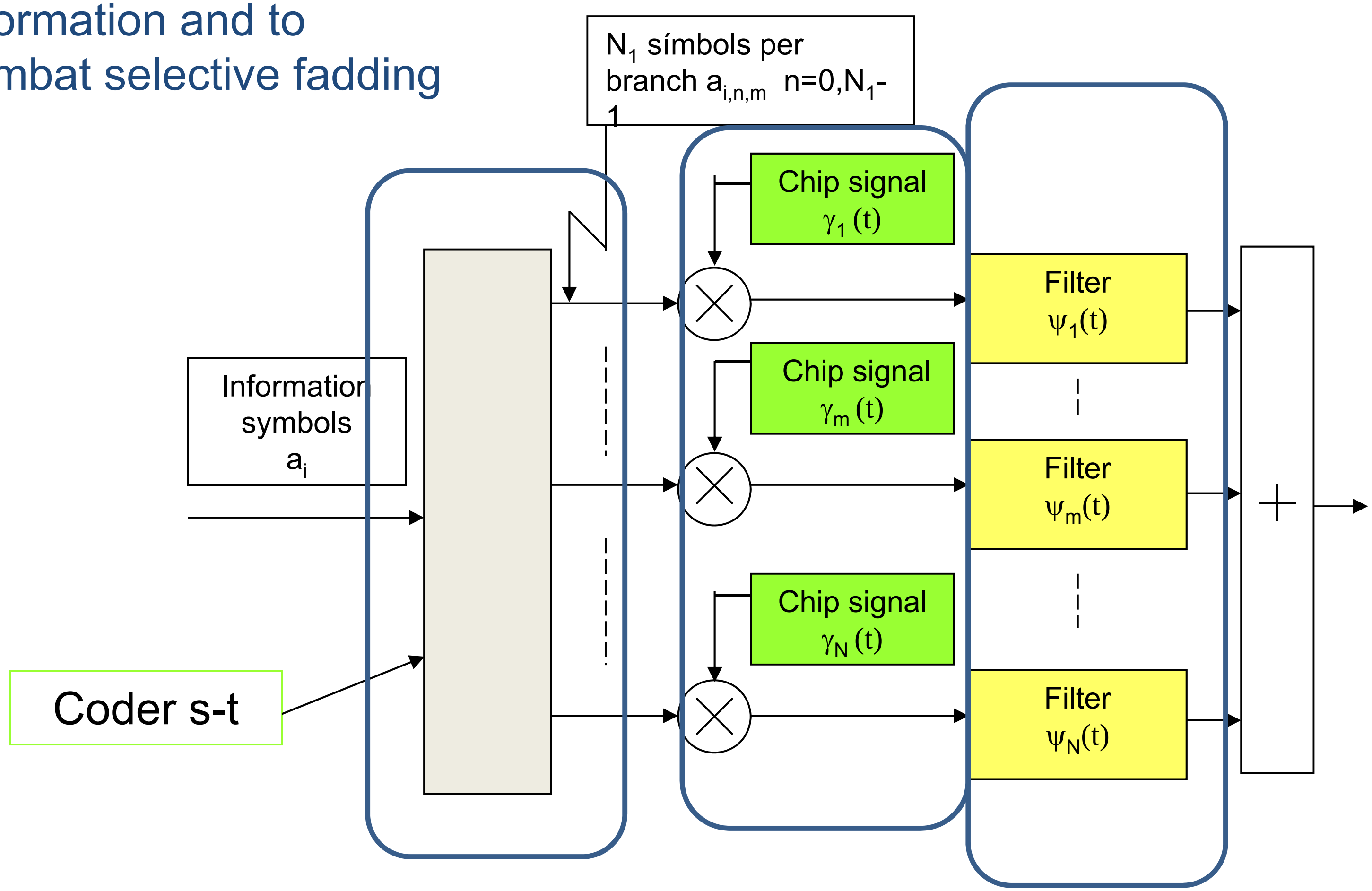
$$\underline{\underline{R}}\underline{a} = (\lambda + 1)^2 \underline{\underline{R}}_c \underline{\underline{R}}^{-1} \underline{\underline{R}}_c^H \underline{a}$$

The Cyclic Beamformer (SCORE)



Beamforming for FDSS

Several levels to protect information and to combat selective fadding



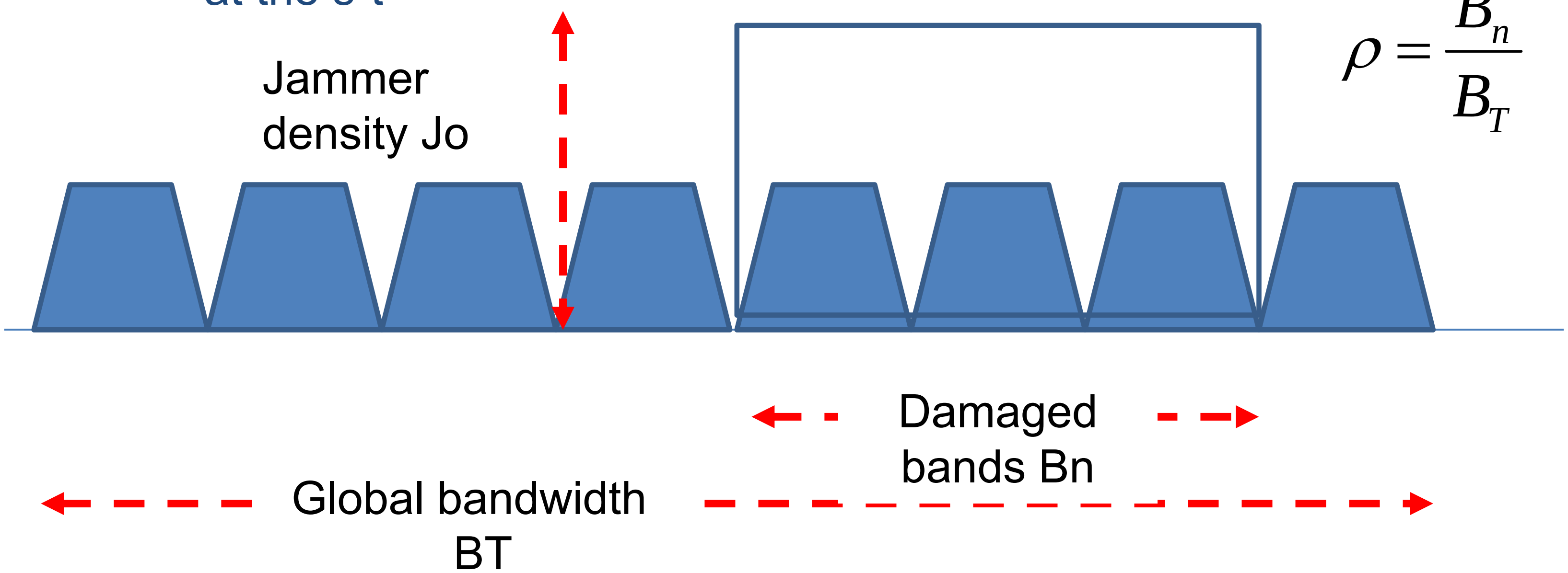
The Tx signal is given by:

$$x_r(t) = \sum_i \sum_{m=1}^{N} a_i \, \gamma_{i,m} \, \Psi_m\left(\frac{t - iT_s}{T_s}\right)$$

Information symbol. Repeated for all branchs (using a block repetition code at the s-t

Chip modulation for every branch

Band pass filter or shaping filter implemented with a Polyphase network



With N bands, the decission variable z will be the adition of the N bands outputs after chip removal and matched filter, where Es is the energy per symbol.

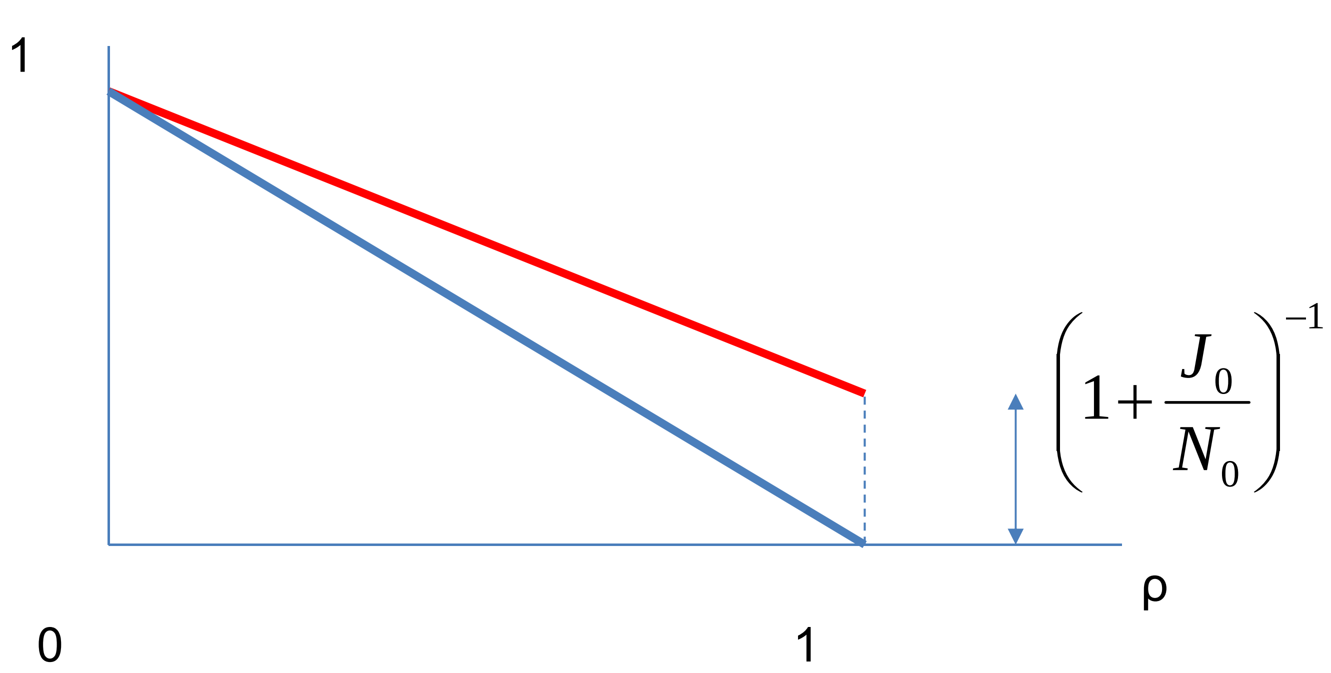
$$z = \sum_{m=1}^{N} (a_i E_s + w_m)$$

The density of the noise will be No for the free bands and Jo+No for the damaged ones.

The SER for the FDSS system is:

$$SER = Q\left(\sqrt{\frac{E_s}{N_0}\left((1-\rho)+\frac{\rho}{\left(1+\frac{J_0}{N_0}\right)}\right)}\right)$$

Jammer Loss for FDSS



A suboptimum receiver is the case when only the free bands are used. In this case, the complexity decreases but also the quality

The goal of beamforming or array processing is to use spatial diversity to decrease the percentage of damaged bands

$$SER = Q\left(\sqrt{\frac{E_s}{N_0}(1-\rho)}\right)$$

At the receiver array, we select two bands: One band free from jamming or interference and other damaged by the jammer.

Clearly, using a repetition code at the s-t coder of the transmitter, the desired will be present in both bands. In consequence, naming Af and Ad the beamformers for each band, the design constrain will be:

$$\underline{a}_f^H \underline{\underline{R}}_{fd} \underline{a}_d + \underline{a}_d^H \underline{\underline{R}}_{df} \underline{a}_f = kte.$$

With this constrain and assuming that both beamformers will produce the desired at their output, the objective will be to minimize the MSE.

$$e(n) = \underline{a}_f^H \underline{X}_{f,n} - \underline{a}_d \underline{X}_{d,n}$$

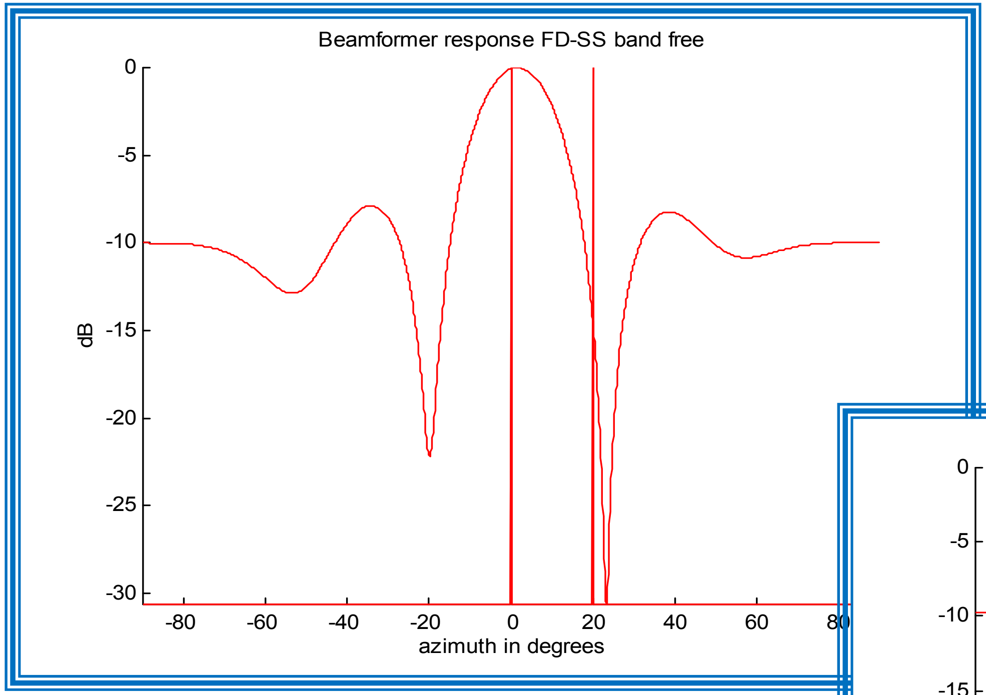
$$\xi = \underline{a}_f^H \underline{\underline{R}}_{ff} \underline{a}_f + \underline{a}_d^H \underline{\underline{R}}_{dd} \underline{a}_d - \underline{a}_f^H \underline{\underline{R}}_{fg} \underline{a}_g - \underline{a}_g^H \underline{\underline{R}}_{gf} \underline{a}_f$$

Solving this objective with the mentioned constrain results on the following solution to the problem:

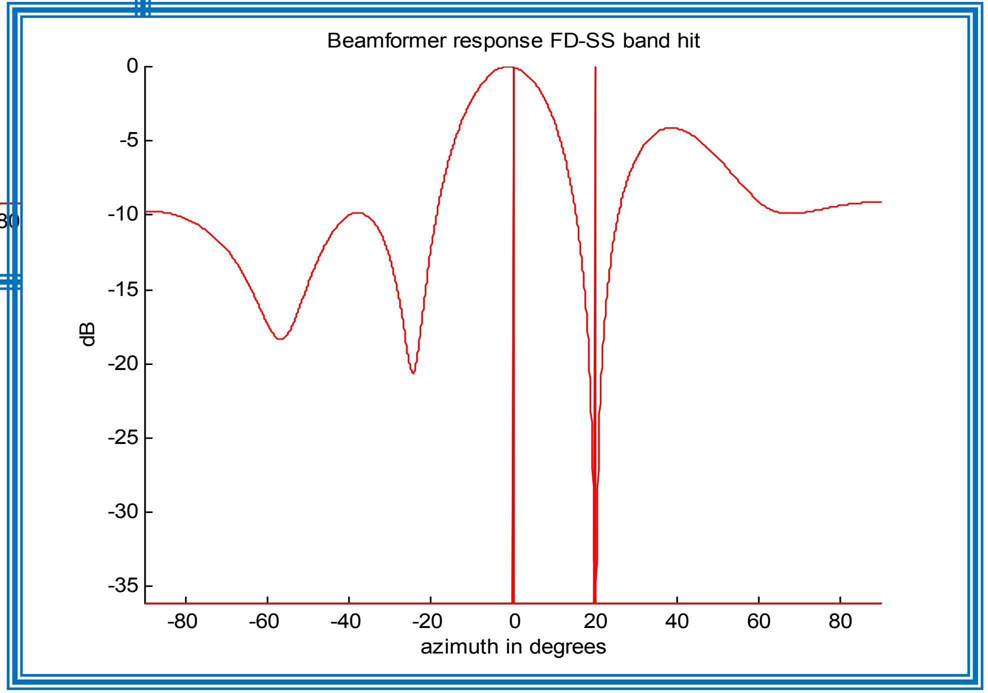
$$\underline{\underline{R}}_{ff} \underline{a}_f = \lambda_{\min} \underline{\underline{R}}_{fd} \underline{\underline{R}}_{dd}^{-1} \underline{\underline{R}}_{df} \underline{a}_f$$

$$\underline{a}_d = \underline{\underline{R}}_{dd}^{-1} \underline{\underline{R}}_{df} \underline{a}_f$$

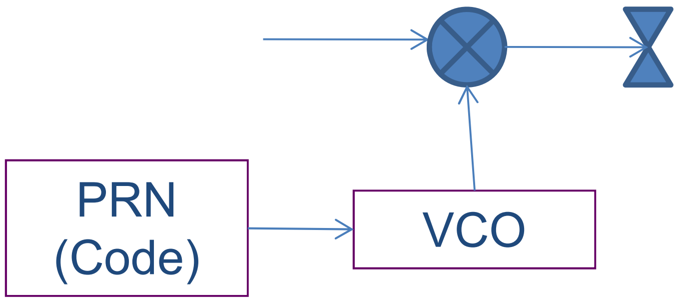
Free band (desired at broadside)



Damaged band (Interferer at 20º)

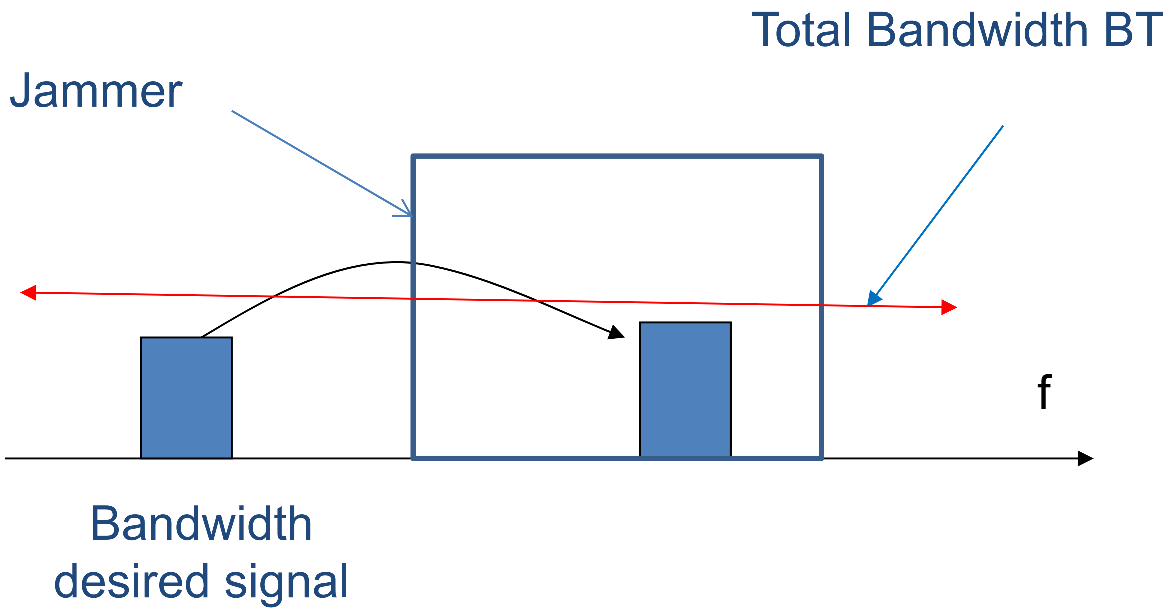


Beamforming for FHSS



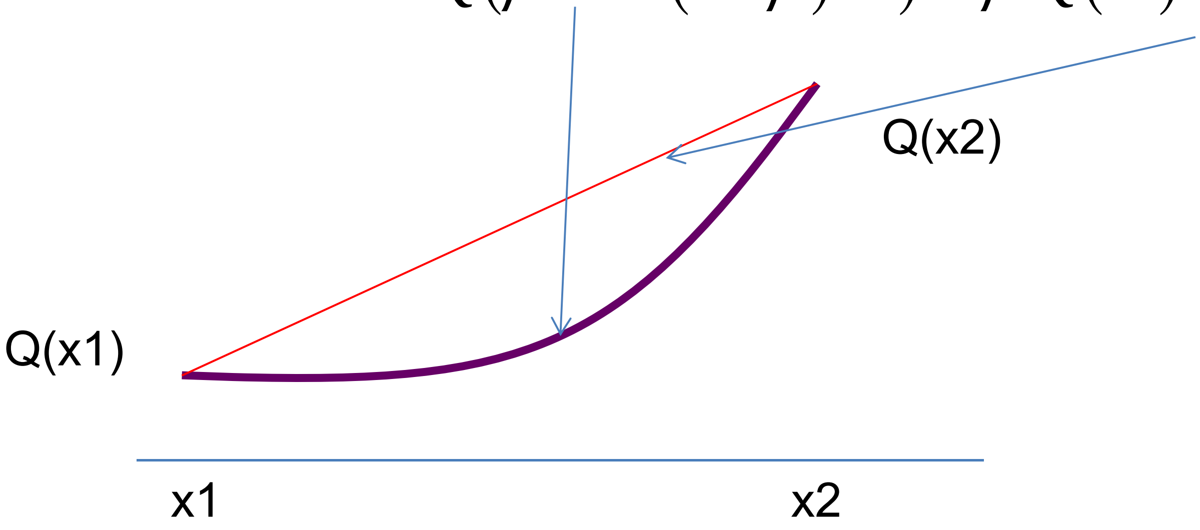
The SER for Coherent (!!!) FH is:

$$SER = (1-\rho)Q\left(\sqrt{\frac{E_s}{N_0}}\right) + \rho\, Q\left(\sqrt{\frac{E_s}{N_0}\left(1+\frac{J_0}{N_0}\right)^{-1}}\right)$$



Since the Q(.) function is concave, it verifies that

$$Q\left(\rho.x1+\left(1-\rho\right)x2\right)\leq \rho.Q\left(x1\right)+\left(1-\rho\right)Q\left(x2\right)$$



In consequence, FDSS is better than FHSS for any damaged percentage, i.e. FDSS is more robust to jamming than FHSS. Furthermore, since coherent hopping is almost impossible the mandatory use of uncoherent FSK in FH deteriorate the system again when compared with FDSS

Again the goal of beamforming for FHSS is to reduce the damage percentage as much as possible

The idea is to compute, by means anticipated processing, the covariance matrix of the next hop, i.e. before the desired hops to the frequency. This covariance is $\underline{\underline{R}}_a$

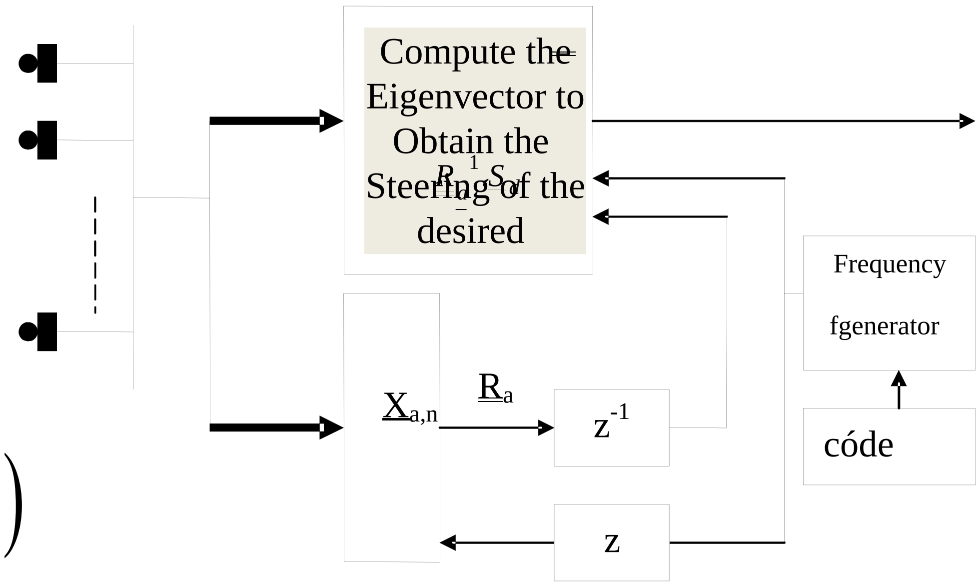
When the desired hops to the frequency, the new covariance is $\underline{\underline{R}}$

since $\underline{\underline{R}} = P_d \underline{S}_d \underline{S}_d^H + \underline{\underline{R}}_a$

Solving the following eigenvalue problem:

$$\underline{\underline{R}}\,\underline{\underline{R}}_a^{-1}\underline{e} = \lambda_{\max}\underline{e}$$

$$\left(P_d \underline{S}_d \underline{S}_d^H \underline{\underline{R}}_a^{-1} + \underline{\underline{I}}\right)\underline{e} = \lambda_{\max}\underline{e}$$

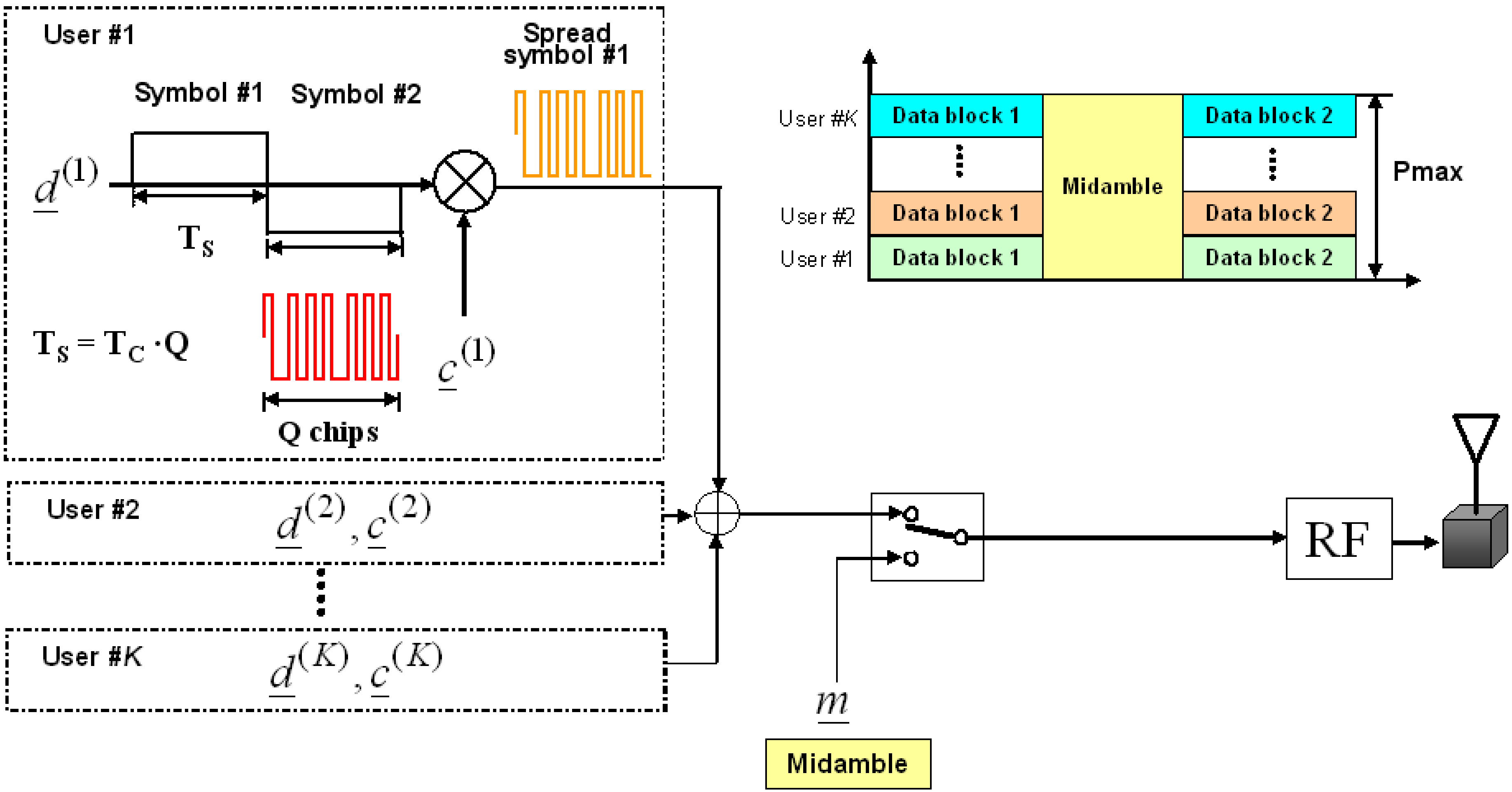


Provides the steering of the desired and the optimum beamformer

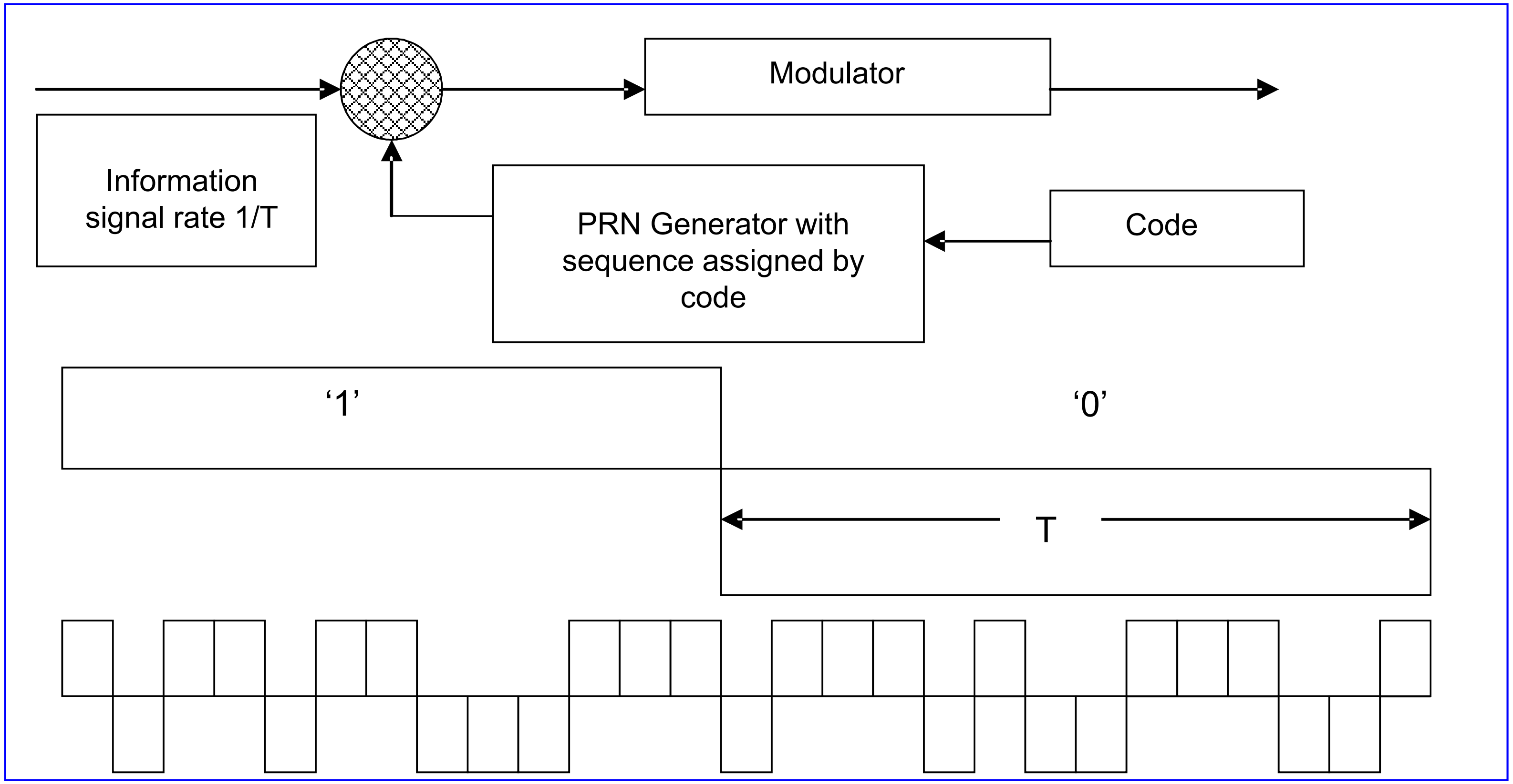
$$\underline{e} = \frac{\underline{S}_d}{\left|\underline{S}_d\right|} \qquad \lambda_{\max} = \left(1 + P_d \underline{S}_d^H \underline{\underline{R}}_a^{-1} \underline{S}_d\right)$$

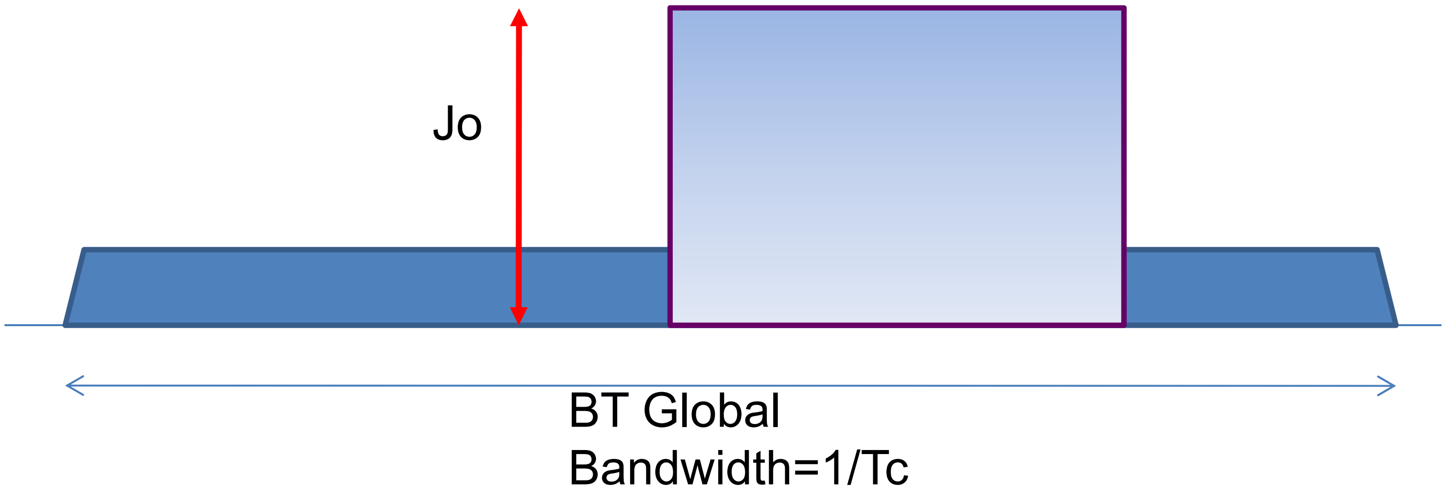
$$\underline{A}_{opt} = \underline{\underline{R}}_a^{-1}\underline{e}$$

DSSS System (CDMA)



DSSS Modulator





After the product by c(t) (the spreading sequence and low pass filter to Bo, the desired original bandwidth, at the receiver

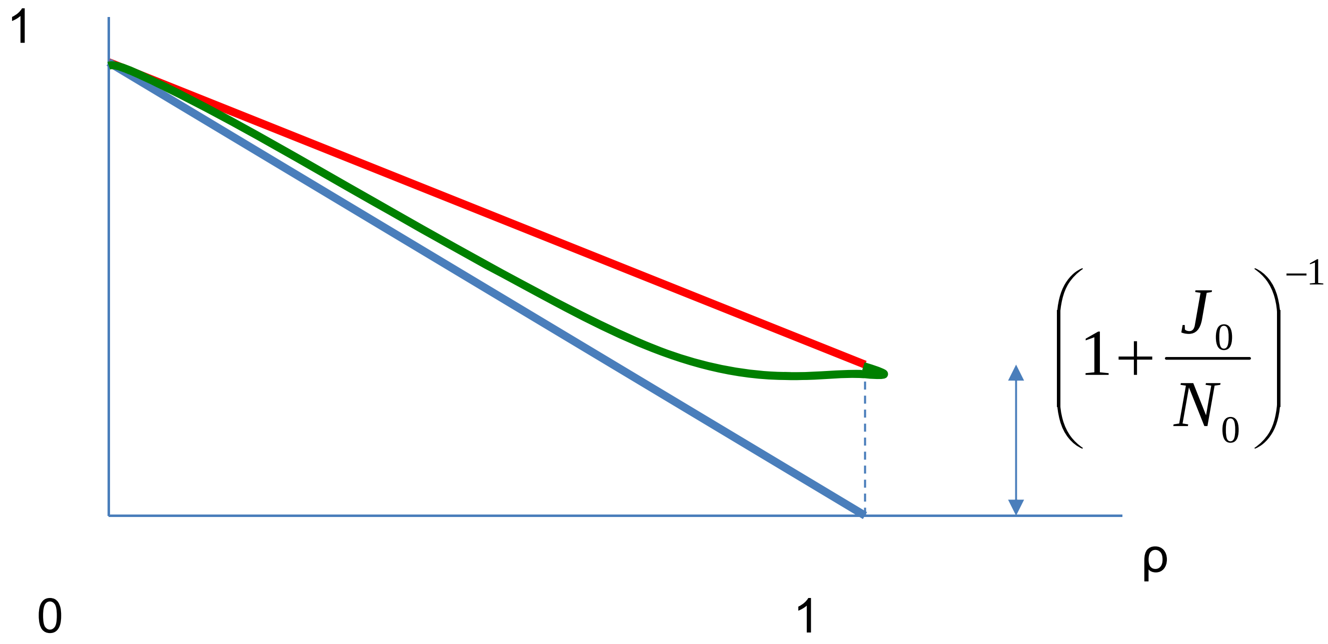
The jammer plus noise density pass to:

$$N_0 + J_0\rho$$

And the SER is:

$$SER = Q\left(\sqrt{\frac{E_s}{N_0}\left(1+\rho\frac{J_0}{N_0}\right)^{-1}}\right)$$

Jammer loss

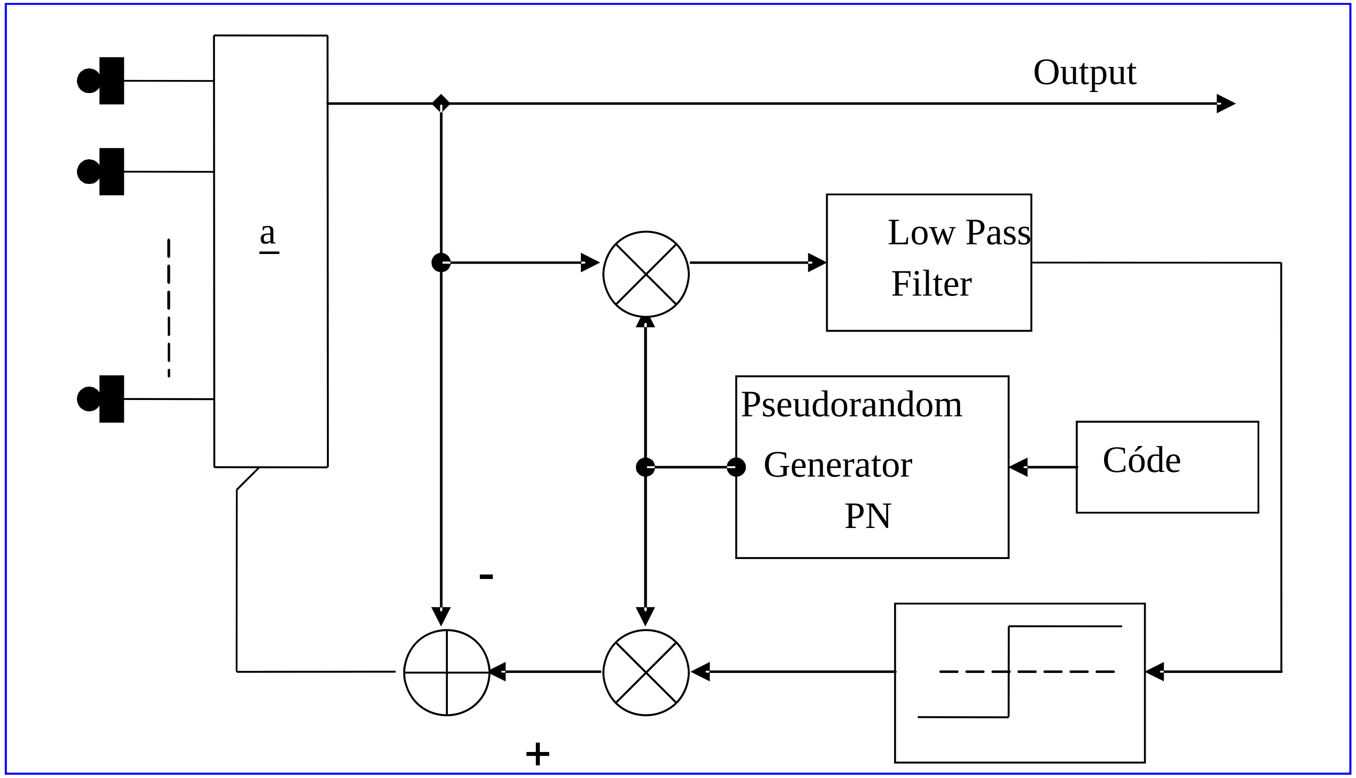


Plot in green shows that DSSS is poorer than FDSS at any percentage level of jamming. In other words OFDM is superior to CDMA in quality.

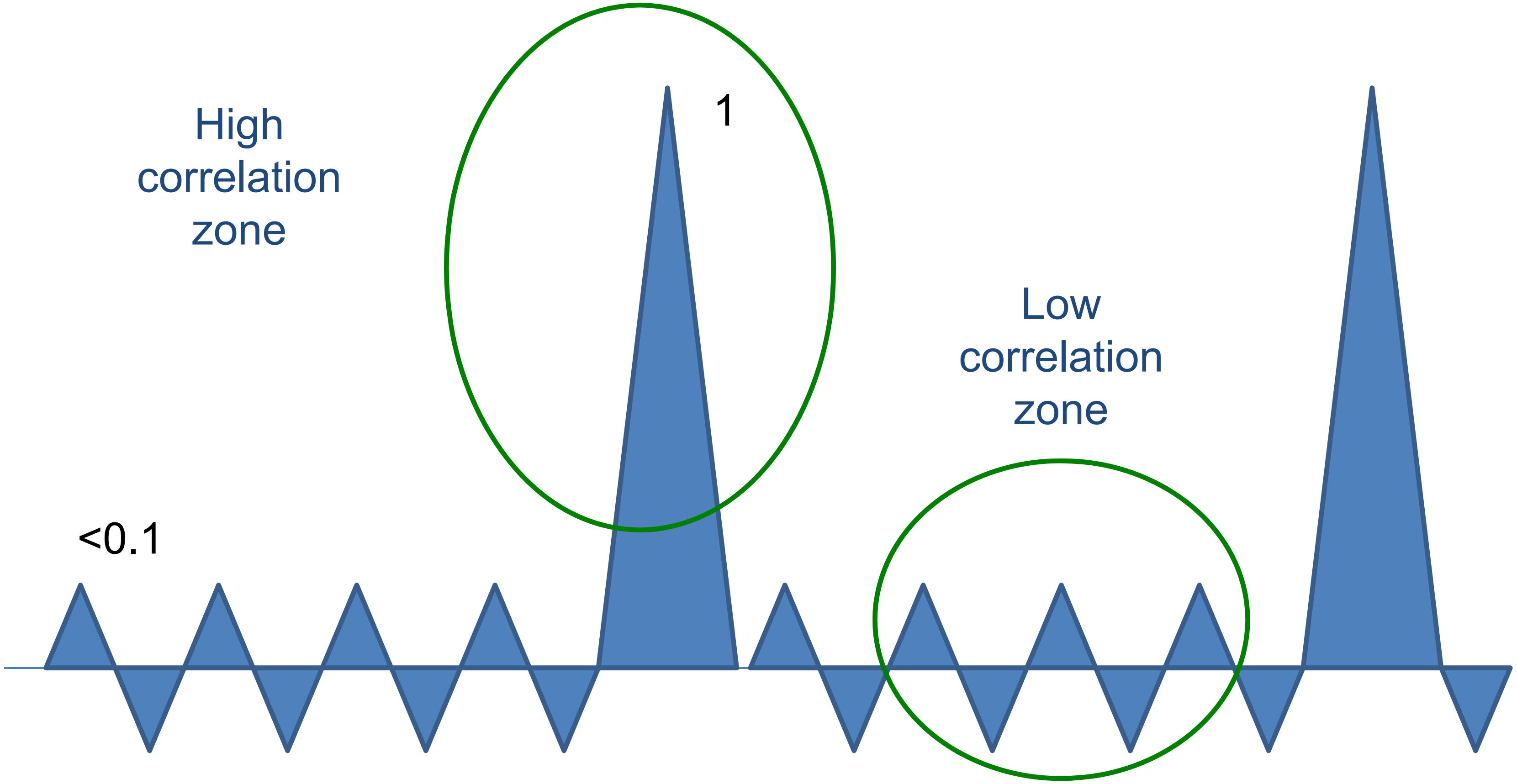
The aim of beamforming again is to reduce the percentage of damage bandwidth. The idea is to locally regenerate the reference. The most difficult part is to get the local spreading sequence in synchronism with the received when beamforming is still not ready. The next figure use a double correlation loop to acquire the mentioned synchronism.

Beamforming for DSSS

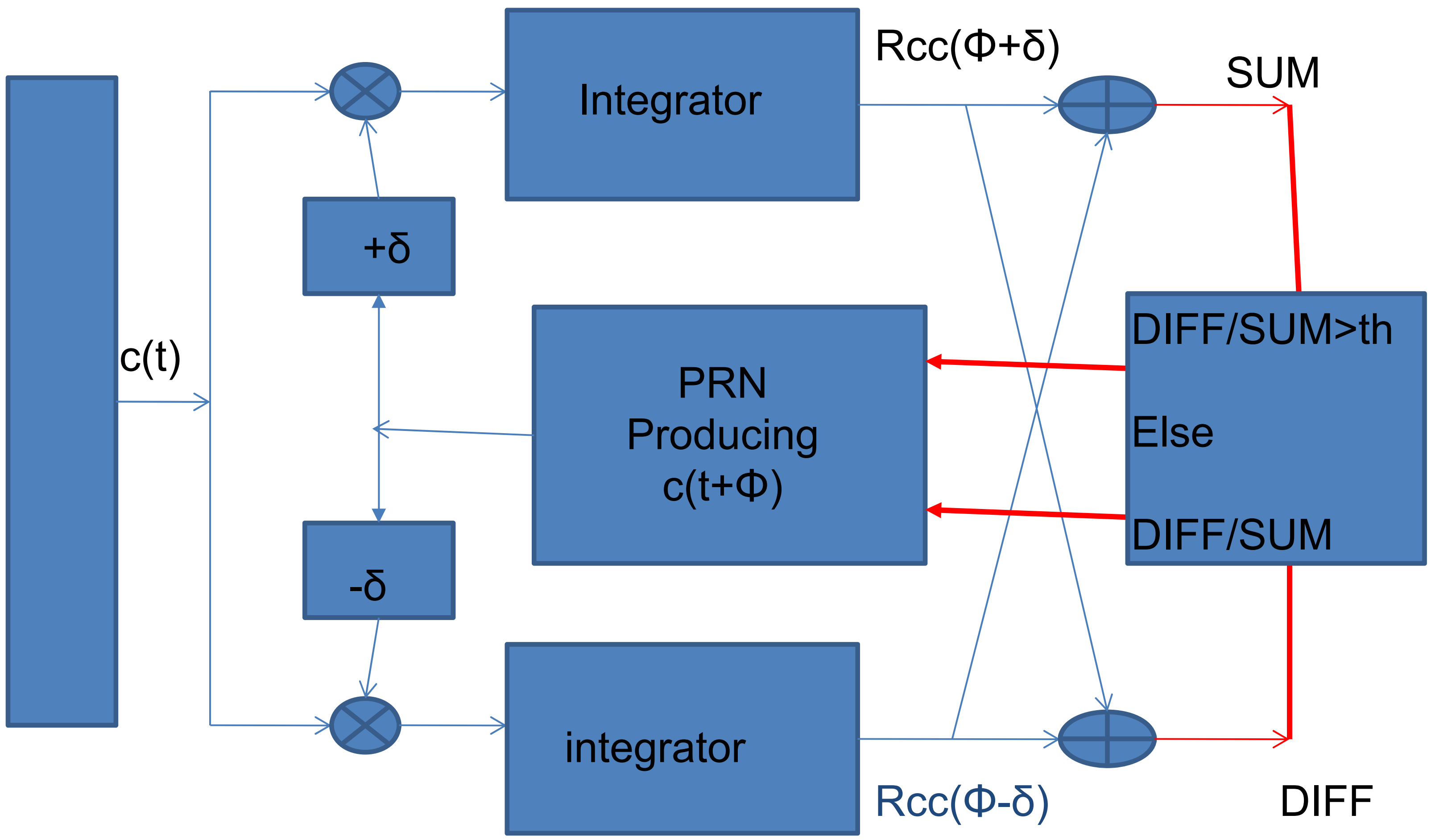
Regeneration of reference (assumes PRN fully synchronized)

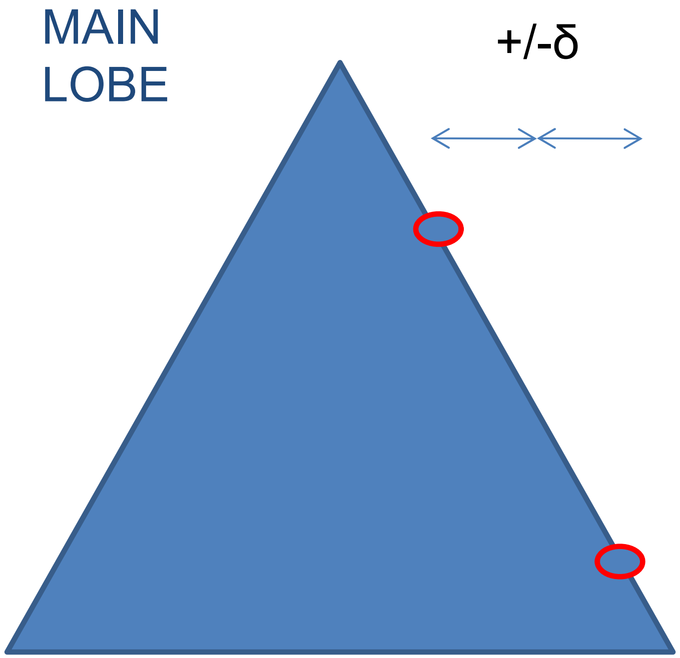


The autocorrelation of c(t)



The Synchronism Loop

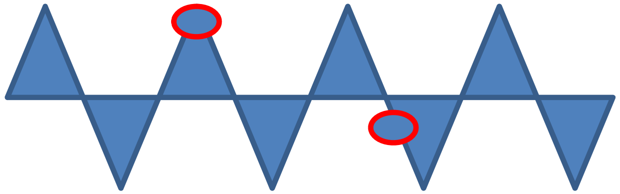




SUM>Threshold

Enters tracking mode. The clock moves in the sense of the difference (normalized by the sum, monopulse tracking)

SECONDARY LOBES



SUM<Threshold

FREE-RUNNING (Move fast delay or advance the clock)

EJERCICIO 1.

Considere un sistema de FDSS con codigo de repeticion (1,M) ancho de banada BT y con simbolos BPSK de energia Es con DOA situado en el broadside de una apertura. El ruido del front-end tiene una densidad espectral No/2 W./Hz.

En el escenario esta presente un jammer situado en un angulo de 60º respecto al broadside con ancho de banda ρ.Bt y densidad espectral Jo/2 W./Hz.

Considere que la apertura receptora, para cada elemento produce una señal zq(n), despues de filtrar en la banda correspondiente, de-scrambling y filtro adaptado. El vector que agrupa todas las señales recibidas de las M bandas viene dado por:

$$\underline{z}_n = a(n).E_s.\underline{1} + \underline{w}_n \quad donde \quad a(n) = \pm 1$$

$$E(\underline{z}_n) = a(n)E_s\underline{1} \quad E(\underline{w}_n.\underline{w}_n^H) = \begin{bmatrix} (J_0 + N_0)\underline{\underline{I}}_{\rho M} & \underline{\underline{0}} \\ \underline{\underline{0}} & N_0\underline{\underline{I}}_{(1-\rho)M} \end{bmatrix}$$

1.- Calcule el estimador ML de a(n) asi como su sesgo y varianza.

2.- Cual es la tasa de error del detector de a(n).

3.- Demuestre cual es el conformador optimo para bandas libres (f) y para bandas dañadas (d).

Siendo

$$\underline{X}_{fn} = a(n)\underline{S}_f + \underline{w}_f$$

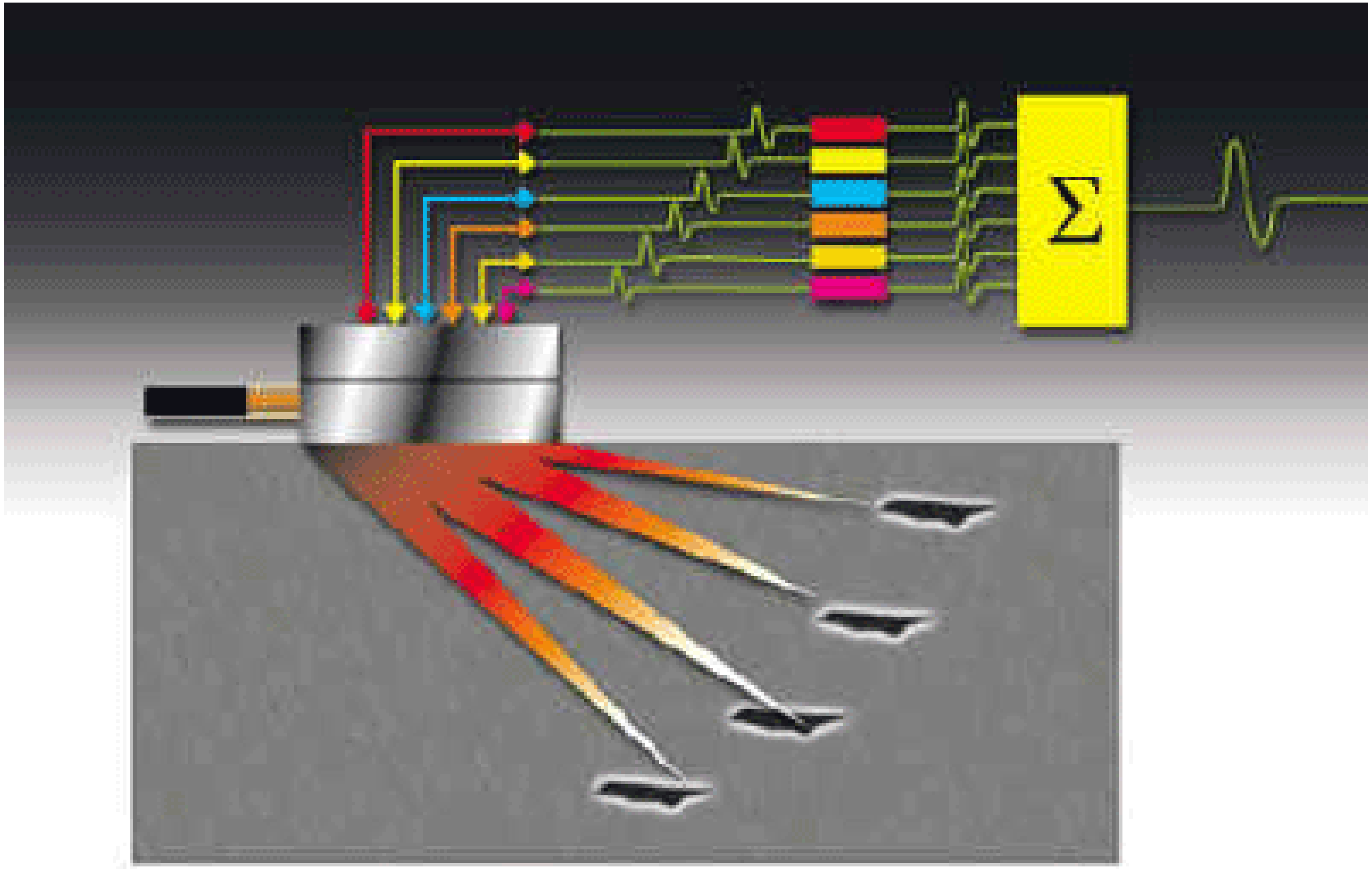
$$\underline{X}_{dn} = a(n)\underline{S}_f + j(n)\underline{S}_j + \underline{w}_f$$

4.- Demuestre que los conformadores para bandas dañadas y bandas libres son respectivamente:

$$\underline{A}_d \propto \underline{\underline{R}}_{dd}^{-1} \underline{S}_f$$

$$\underline{A}_f \propto \underline{S}_f$$

5.- Que ventaja representa el uso de secuencias de scrambling diferentes para cada banda de frecuencia.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