The Centre Tecnològic de Telecomunicacions de Catalunya

A gateway to advanced communication technologies

LOW COMPLEXITY MIMO: Non Cooperative MIMO-MAC and MIMO-BC

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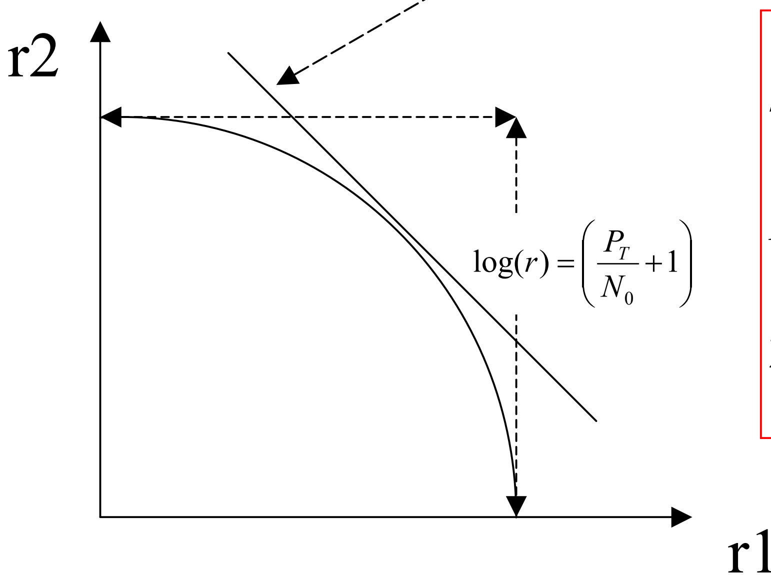
CAPACITY Non-Cooperative

Capacity for a cooperative MIMO with ideal channel (i.e. Identity matrix) is:

$$C_{SUM}^{coop} = \log\left(1+\frac{P_1}{N_0}\right) + \log\left(1+\frac{P_2}{N_0}\right)$$

$$C_{SUM}^{coop} = r1 + r2$$

Since $P_1 + P_2 = P_T$



$$r1=\log\left(1+\frac{P_1}{N_0}\right) \quad r2=\log\left(1+\frac{P_2}{N_0}\right)$$

$$P_1+P_2=E_T$$

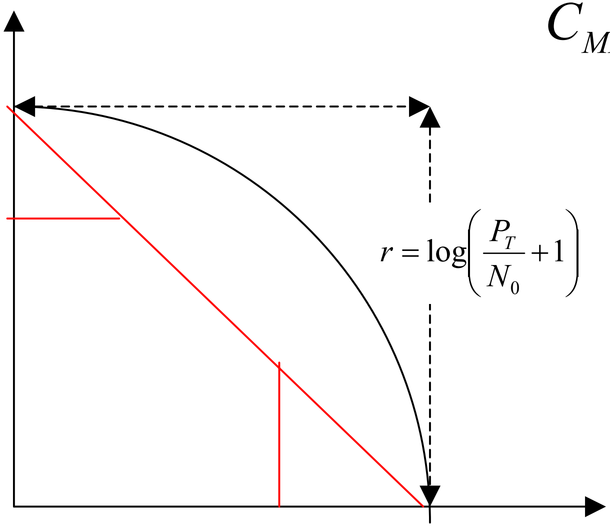
$$2^{r1}+2^{r2}=\left(2+\frac{P_T}{N_0}\right)$$

MIMO-MAC

The two users canot cooperate at Rx

$$C_{MIMO-MAC}^{Not\,optimum}=\log\left(1+\frac{P_1}{P_2+N_0}\right)+\log\left(1+\frac{P_2}{P_1+N_0}\right)$$

DP (Costa).- The best receiver decodes, let us say, user 1 considering user 2 as interference. Then Tx, substracts user 1 from the signal sent to user 2.



$$C_{MIMO-MAC}=\log\left(1+\frac{P_1}{P_2+N_0}\right)+\log\left(1+\frac{P_2}{N_0}\right.$$

$$r1=\log\left(1+\frac{P_1}{P_2+N_0}\right)\qquad r2=\log\left(1+\frac{P_2}{N_0}\right)$$

$$P_1+P_2=E_T$$

$$r1+r2=\log\left(1+\frac{P_T}{N_0}\right)$$

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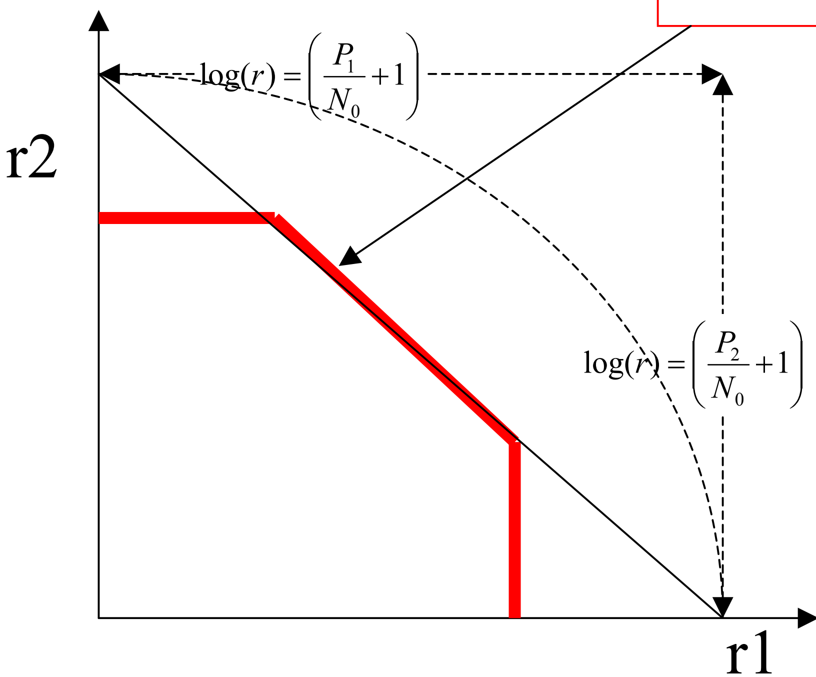
MIMO-MAC

No cooperation at Tx, receiver with succesive interference cancelation.

$$r1 = \log\left(1 + \frac{P_1}{P_2 + N_0}\right) \qquad r2 = \log\left(1 + \frac{P_2}{N_0}\right)$$

$$r1 + r2 = \log\left(1 + \frac{P_1 + P_2}{N_0}\right)$$



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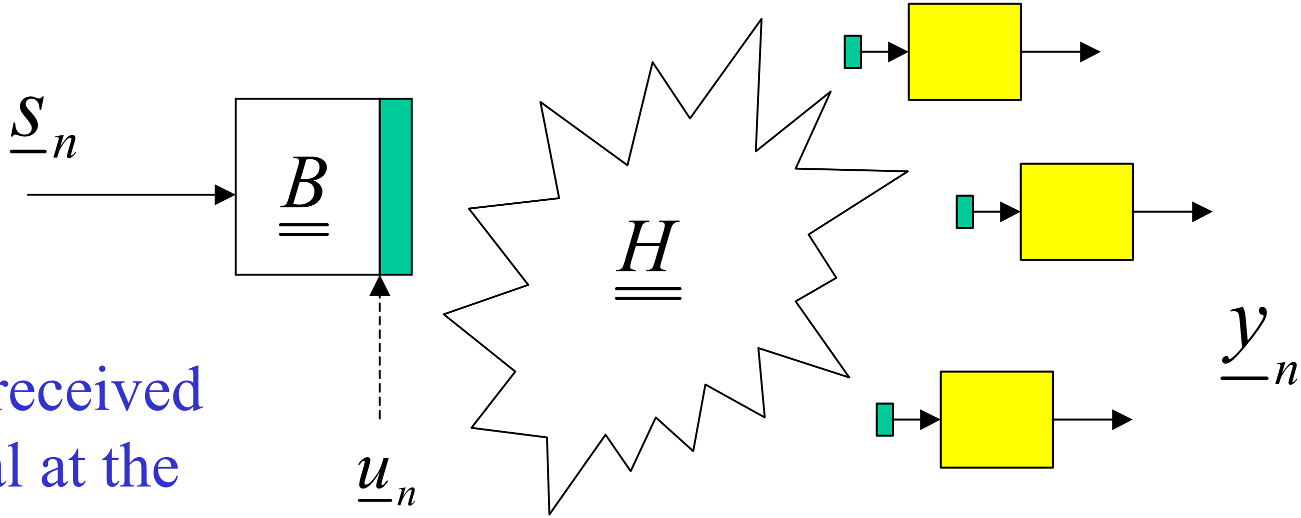
The Zero Forcing MIMO-BC



The received signal at the set of users is:

$$\underline{y}_n = \underline{\underline{H}}.\underline{\underline{B}}.\underline{s}_n + \underline{w}_n$$

$$\underline{\underline{B}}_0 = \left(\underline{\underline{H}}^H.\underline{\underline{H}}\right).\underline{\underline{H}}$$

Constraining the Tx power

$$\underline{\underline{B}}_{ZF} = \alpha.\underline{\underline{B}}_0$$

$$\alpha = \frac{P_T}{tr\left(\underline{\underline{B}}_0.\underline{\underline{B}}_0^H\right)}$$

The problem is $E(\alpha) \to \infty$

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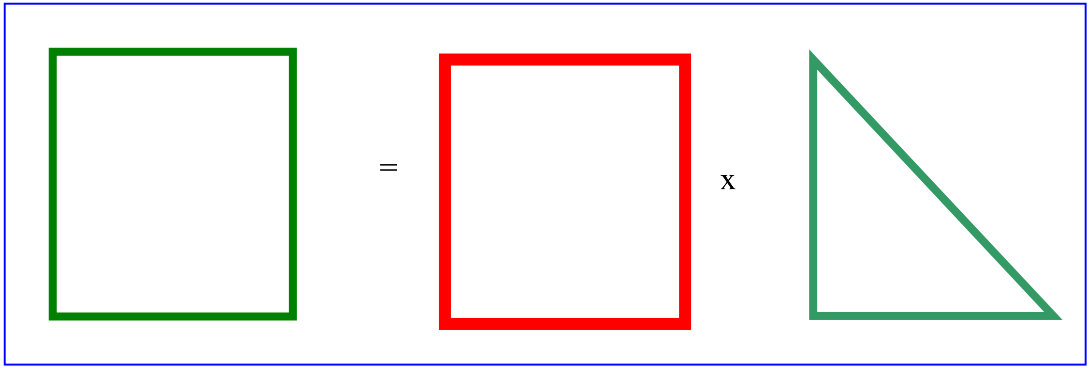
The QR Decomposition

$$\underline{\underline{H}} = \underline{\underline{Q}}.\underline{\underline{R}}$$

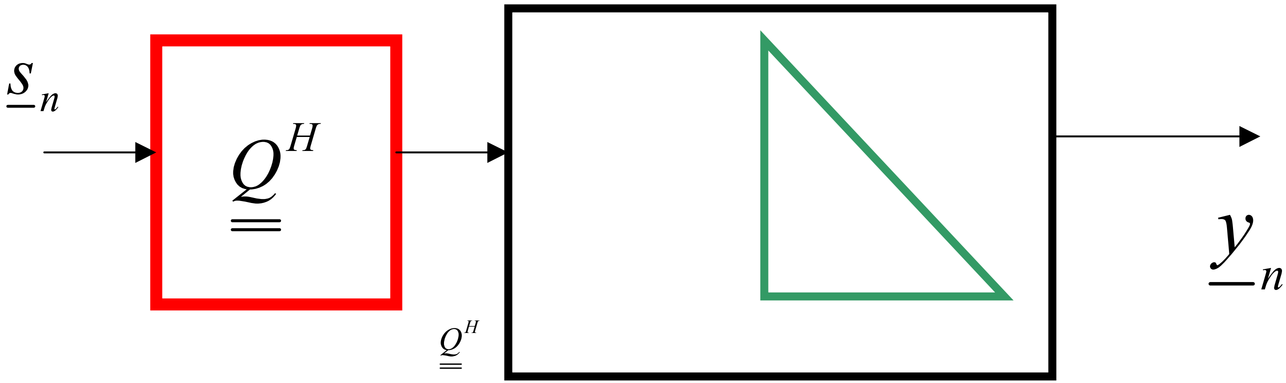


with

$$\underline{\underline{B}} = \underline{\underline{Q}}^{H}$$

The MIMO chanel reduces to lower triangular. This is the link with the DP implementation (degraded channel)

Successive Cancellation



The system reduces to:

$$\begin{bmatrix} y1 \\ y2 \\ y3 \\ .. \\ yN \end{bmatrix} = \begin{bmatrix} r11 & 0 & 0 & .. & 0 \\ r12 & r22 & 0 & .. & 0 \\ r13 & r23 & r33 & .. & 0 \\ .. & .. & .. & .. & .. \\ r1N & r2N & r3N & .. & rNN \end{bmatrix} . \begin{bmatrix} s1 \\ s2 \\ s3 \\ .. \\ sN \end{bmatrix} + \begin{bmatrix} w1 \\ w2 \\ w3 \\ .. \\ wN \end{bmatrix}$$

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$$s1 \approx dec\left(y1/r11\right)$$

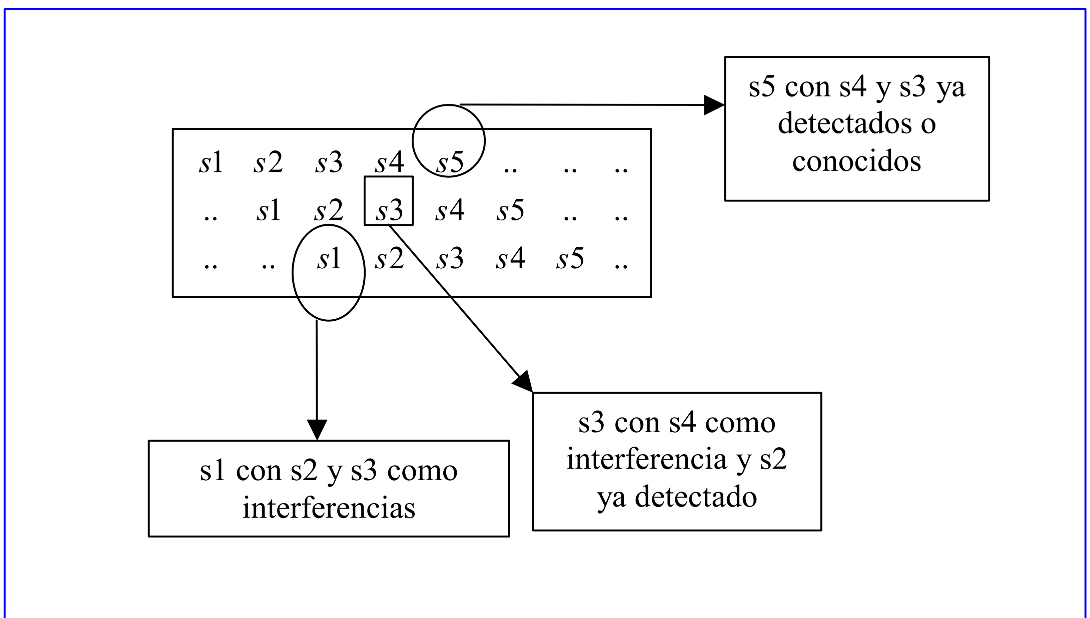
$$s2 \approx dec\left(\left(\frac{y2}{r22}\right) - \left(\frac{r12}{r22}\right).s1\right)$$

....

$$sN \approx dec\left(\left(\frac{yN}{rNN}\right) - \sum_{q=1}^{N-1}\left(\frac{rqN}{rNN}\right).sq\right)$$

Subtracting inteference:
Succesive interf. cancellation

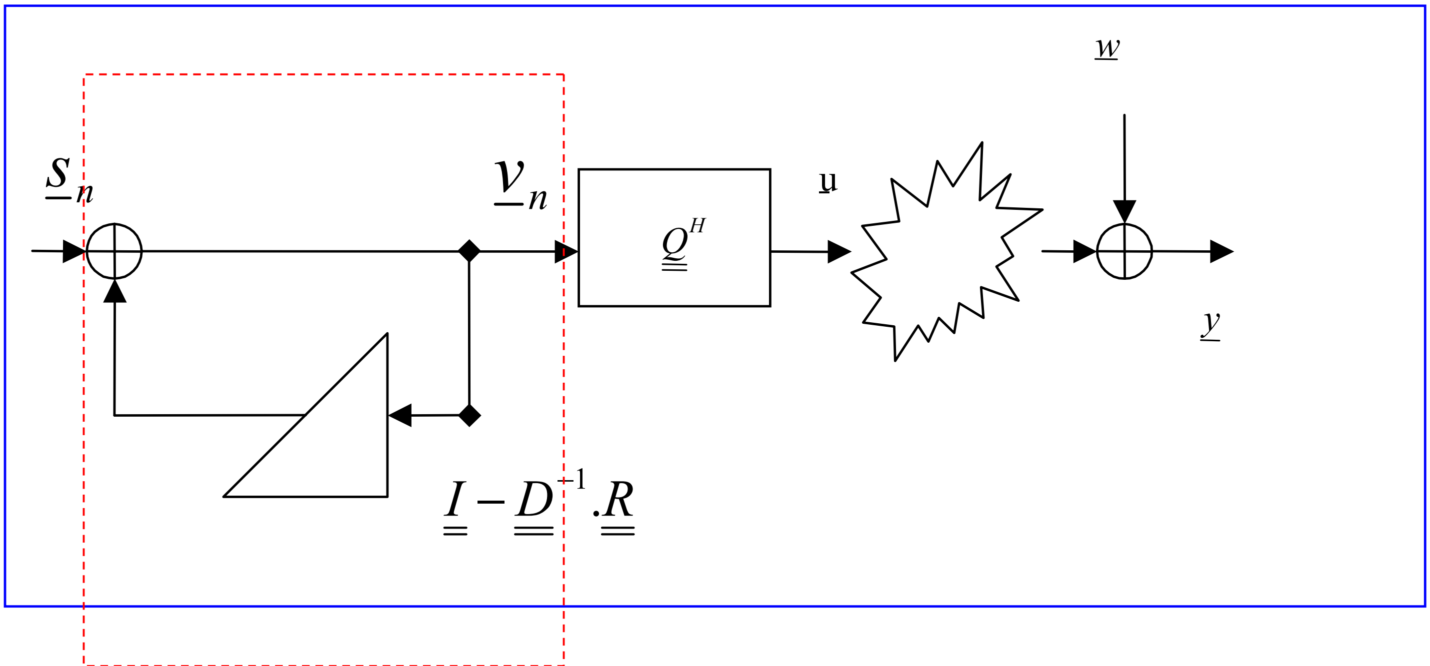
Each symbol is interfered only from the upper layers previously detected



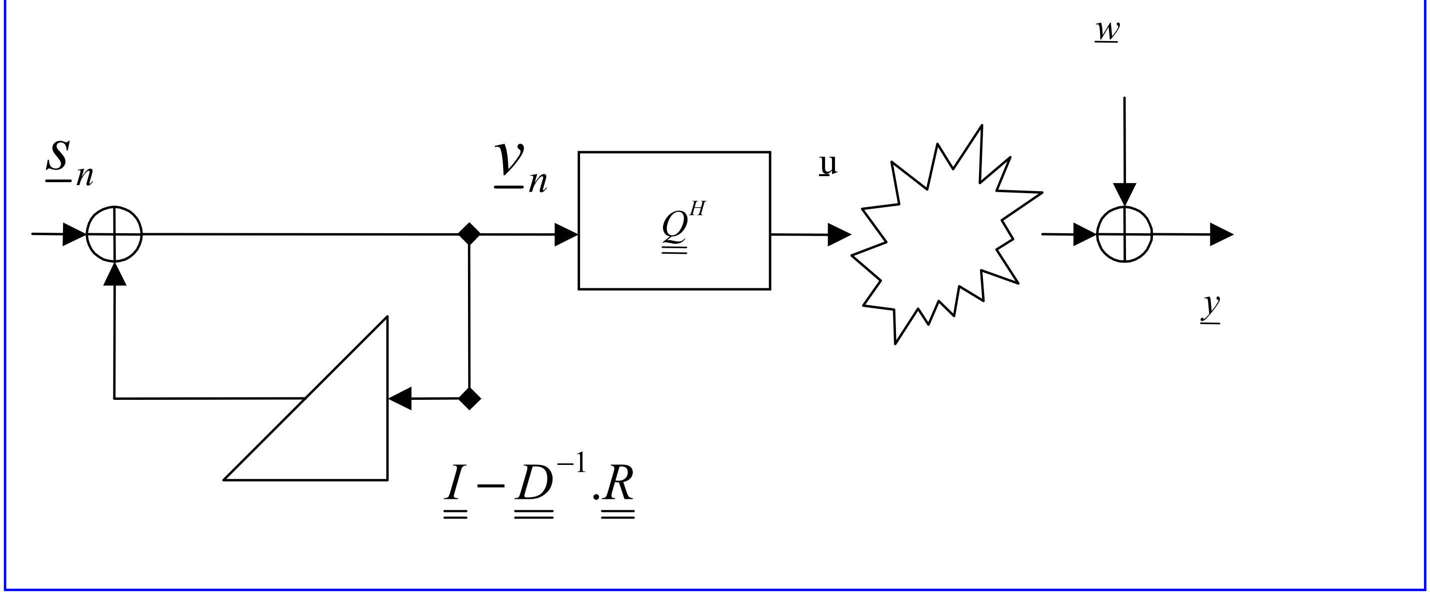
BLAST

MIMO-BC: Pre-subtracted intereference

with $\underline{\underline{D}} = diag\left[\underline{\underline{R}}\right]$



This implements just the diagonalizacion of matrix R (the product of this loop by R is equal to diag(R)).



$$\underline{v} = \underline{s} + \left(\underline{\underline{I}} - \underline{\underline{D}}^{-1}.\underline{\underline{R}}\right).\underline{v}$$

$$v1 = s1$$

$$v2 = s2 - \left(\frac{r12}{r22}\right).s1$$

....

The interference is a-priori removed at the transmitter

$$y1 = r11.s1 + w1$$

$$y2 = r22.s2 + w2$$

....

The problem is to control the Tx power (including this recursive operation)

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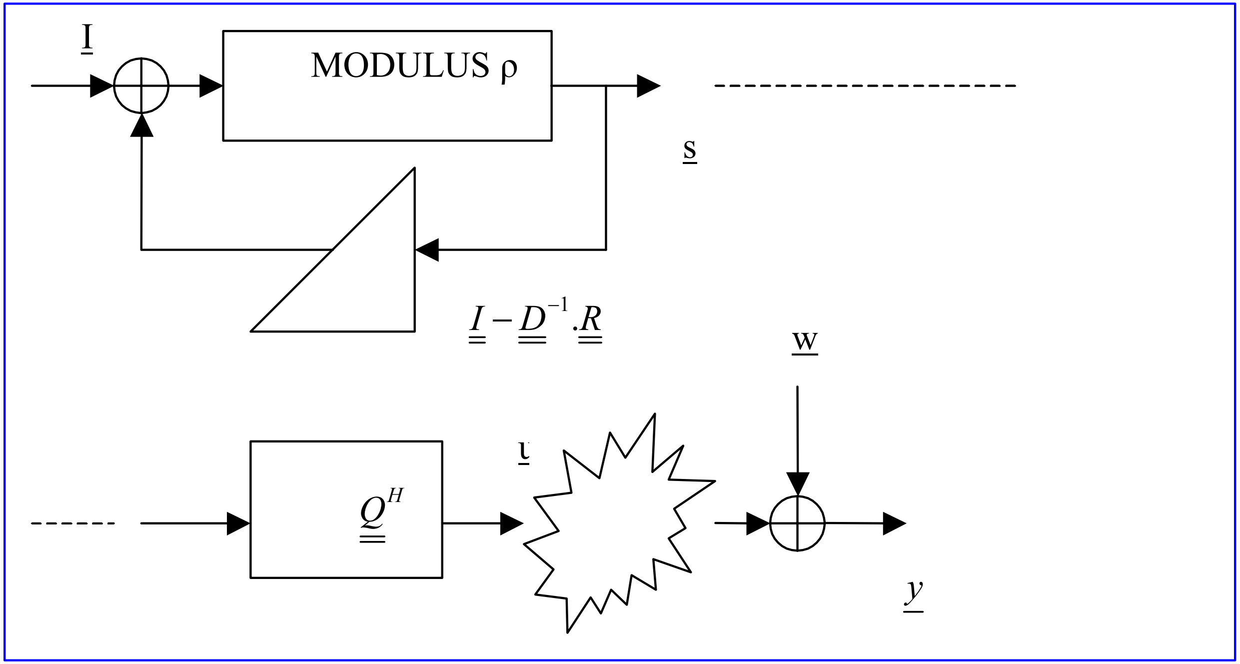
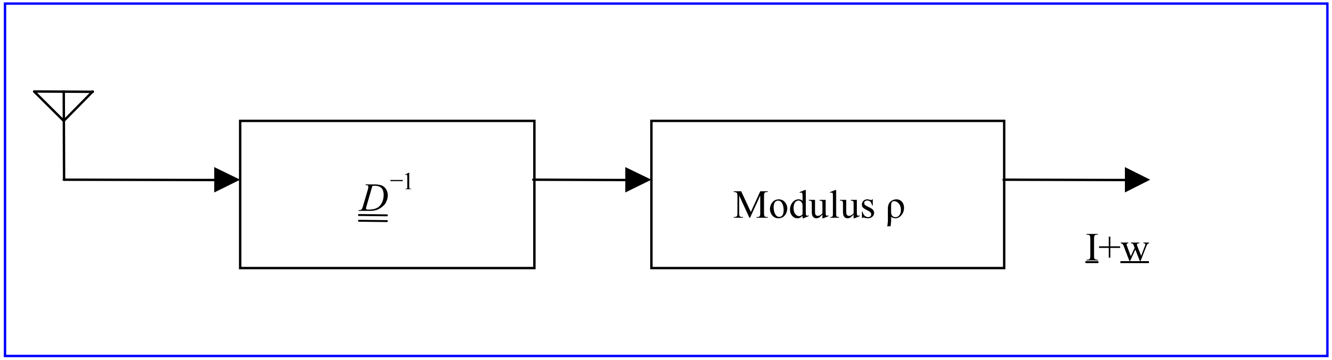
The T-H precoder (Partial Response Communications)



The Tx power is bounded to:

$$E_T = n_S \cdot \frac{\rho^2}{12}$$

$$\underline{s} = MOD_\rho\left(\underline{I} + \left(\underline{\underline{I}} - \underline{\underline{D}}^{-1}.\underline{\underline{R}}\right).\underline{s}\right) = \underline{I} + \left(\underline{\underline{I}} - \underline{\underline{D}}^{-1}.\underline{\underline{R}}\right).\underline{s} + \underline{m}.\rho$$



Since D is diagonal, every user can detect independently, without interference its own signal

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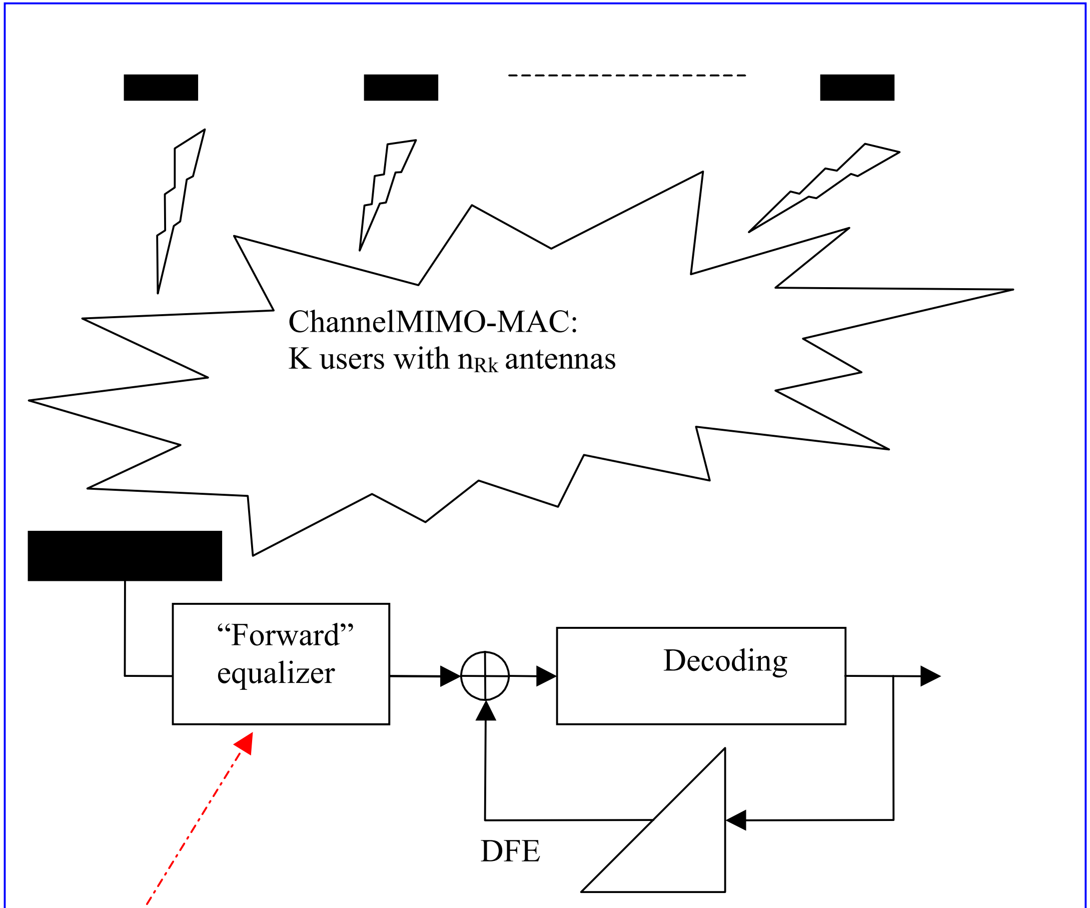
MIMO-MAC



$\underline{\underline{Q}}$ matrix processing

Sphere Pre-coder

In general, the Tx power control can be described as:

$$\underline{s} = \frac{1}{\sqrt{\gamma}} \underline{\underline{B}}.\underline{I} \qquad \gamma = Traza\left(\underline{\underline{B}}.\underline{I}.\underline{I}^H.\underline{\underline{B}}^H\right) = \left|\underline{\underline{B}}.\underline{I}\right|_F^2$$

$$\underline{I} \quad \Rightarrow \quad \underline{I} + \rho.\underline{m}$$

Modifies the Tx symbol with an integer operation which can be removed by a modulus ρ operation

The Tx vector is:

$$\underline{s} = \frac{1}{\sqrt{\gamma}}.\underline{\underline{B}}.\left(\underline{I} + \rho.\underline{m}\right)$$

The Tx design is to find the optimum integer vector in a similar manner it is done in a ML decoder

$$\underline{m} = \min_{\underline{m}'} \left\| \underline{\underline{B}}.\left(\underline{I} + \rho.\underline{m}'\right)\right\|_F^2$$

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The MSE version

In a MSE receiver

$$\hat{\underline{s}} = \underline{\underline{A}}_{MSE}^{H}.\underline{X}_{R} = \underline{\underline{A}}_{MSE}^{H}.\underline{\underline{H}}.\underline{s} + \underline{\underline{A}}_{MSE}^{H}.\underline{w} = \underline{s} - \underline{e}$$

This is the MSE

then

$$\underline{e} = \left(\underline{\underline{I}} - \underline{\underline{A}}_{MSE}^{H}.\underline{\underline{H}}\right)\underline{s} + \underline{\underline{A}}_{MSE}^{H}.\underline{w}$$

IN ORDER TO ALLOW INDEPENDENT `PROCESSING AT THE USERS' RECEIVERS THE MSE MATRIX MUST BE DIAGONAL

$$\underline{\underline{R}}_{e} = \left(\underline{\underline{I}} - \underline{\underline{A}}^{H}.\underline{\underline{H}}\right)\left(\underline{\underline{I}} - \underline{\underline{A}}^{H}.\underline{\underline{H}}\right)^{H} + \underline{\underline{A}}^{H}.\underline{\underline{A}} =$$

$$= \left(\underline{\underline{A}} - \left(\underline{\underline{I}} + \underline{\underline{H}}.\underline{\underline{H}}^{H}\right)^{-1}.\underline{\underline{H}}\right)^{H}..\left(\underline{\underline{A}} - \left(\underline{\underline{I}} + \underline{\underline{H}}.\underline{\underline{H}}^{H}\right)^{-1}.\underline{\underline{H}}\right) +$$

$$+ \left[\underline{\underline{I}} - \underline{\underline{H}}^{H}.\left(\underline{\underline{I}} + \underline{\underline{H}}.\underline{\underline{H}}^{H}\right)^{-1}.\underline{\underline{H}}\right]$$

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In consequence (using the inverse lemma for the MSE matrix):

$$\underline{\underline{A}}_{MSE} = \left(\underline{\underline{I}} + \underline{\underline{H}}.\underline{\underline{H}}^H\right)^{-1}.\underline{\underline{H}} = \underline{\underline{R}}_e^{-1}.\underline{\underline{H}}$$

$$\underline{\underline{R}}_e^{MSE} = \left(\underline{\underline{I}} + \underline{\underline{H}}.\underline{\underline{H}}^H\right)^{-1}$$

Let us assume that $\underline{\underline{R}}_e^{MSE} = \left(\underline{\underline{I}} + \underline{\underline{H}}.\underline{\underline{H}}^H\right)^{-1} = \left(\underline{\underline{G}}.\underline{\underline{S}}.\underline{\underline{G}}^H\right)^{-1}$

$$\underline{s} = \underline{\hat{s}} + \underline{e} = \underline{\underline{A}}^H.\underline{X}_R + \underline{e}$$

$$\underline{\underline{G}}.\underline{s} = \underline{\underline{G}}.\underline{\underline{A}}^H.\underline{X}_R + \underline{\underline{G}}.\underline{e}$$

add $\underline{s}$ *both sides*

$$\underline{s} = \left(\underline{\underline{I}} - \underline{\underline{G}}\right).\underline{s} + \underline{\underline{B}}_{MSE}.\underline{X}_R + \underline{\underline{G}}.\underline{e}$$

Define B_{MSE}

This error has diagonal covariance

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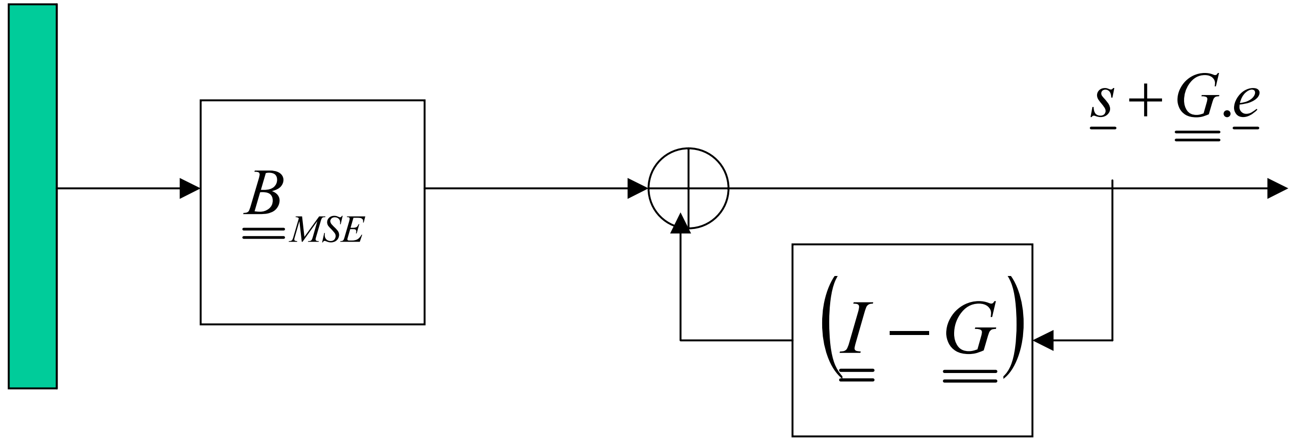

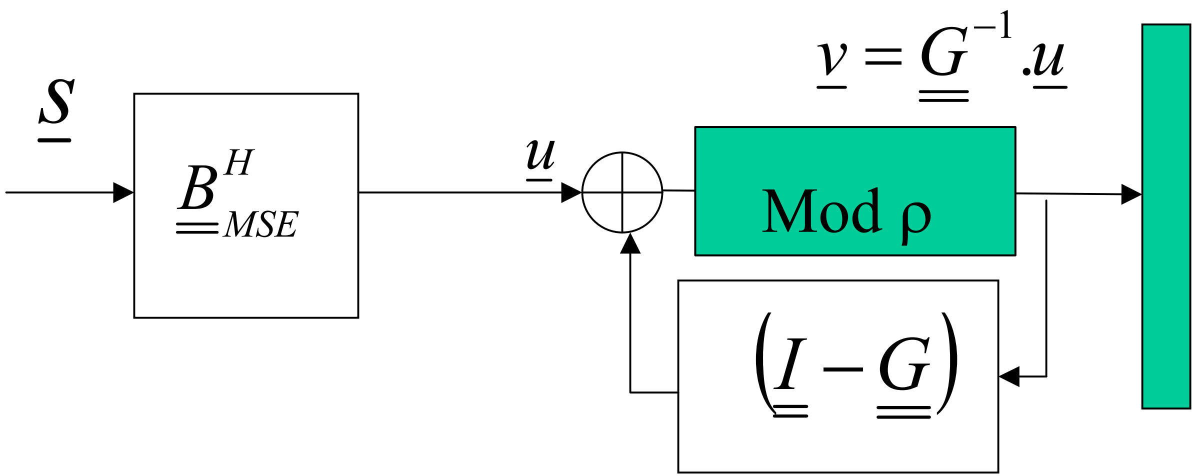

$$\underline{X}_R = \underline{\underline{H}}.\underline{\underline{G}}^{-1}.\underline{\underline{B}}_{MSE}.\underline{s} + \underline{w} = \underline{\underline{H}}.\underline{\underline{G}}^{-1}.\underline{\underline{G}}.\underline{\underline{A}}^H_{MSE}.\underline{s} + \underline{w}$$

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