

innovating communications

The Centre Tecnològic de Telecomunicacions de Catalunya

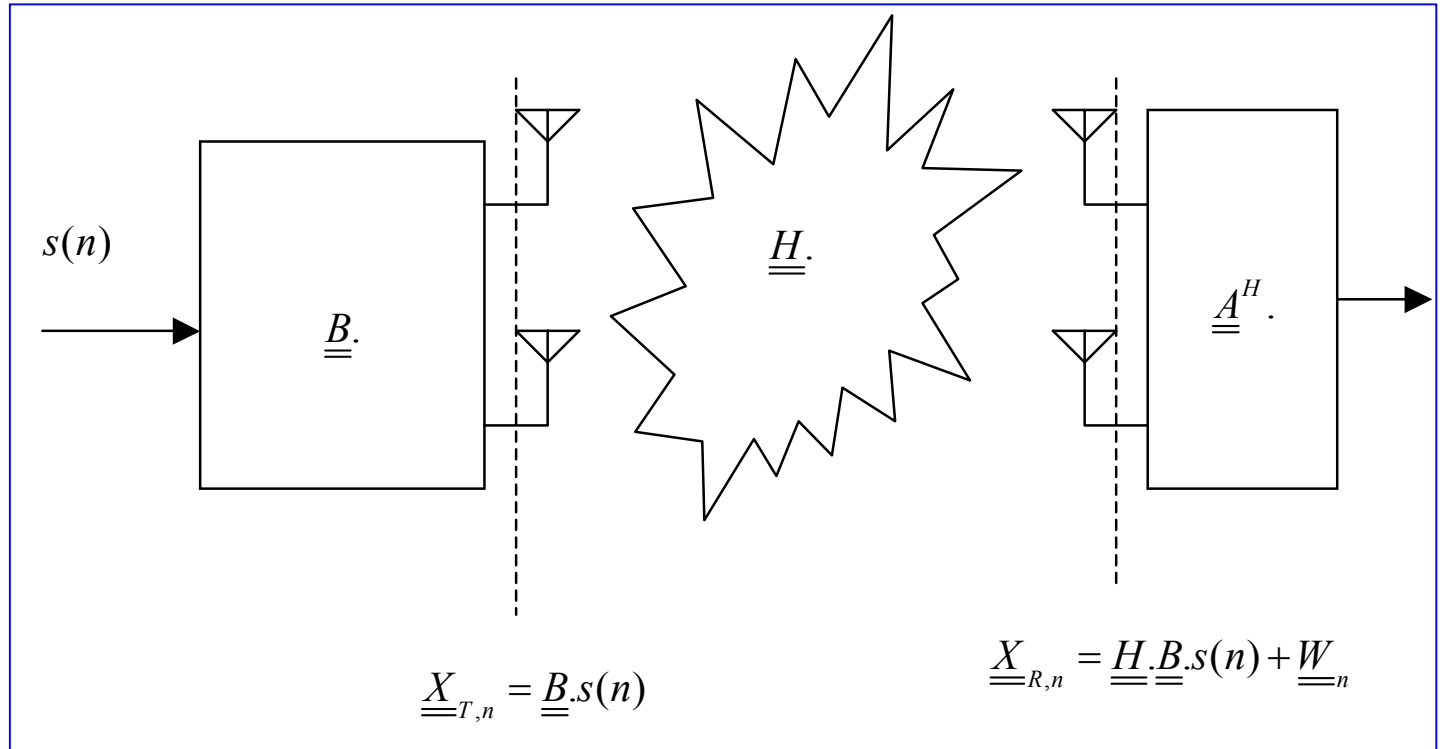
A gateway to advanced communication technologies

**MIMO1: Single symbol over flat
fading CSIT/CSIR**

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The Scenario



Flat Fading, CSIT and CSIR, Single Symbol

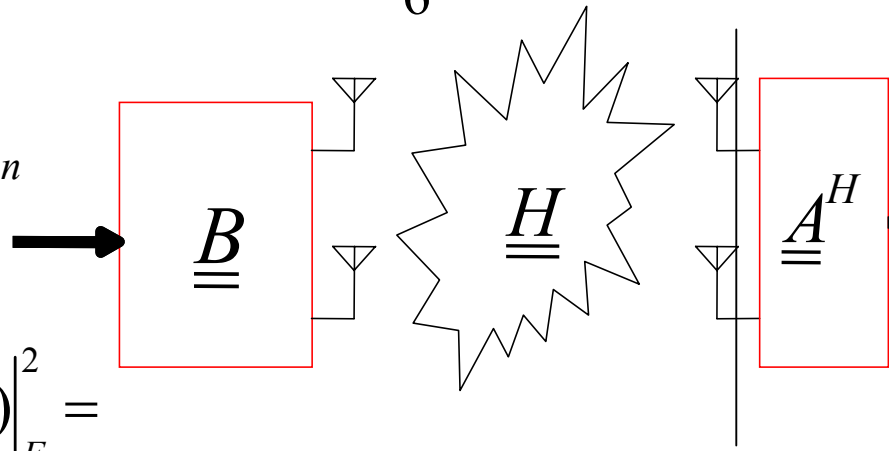
The ML Receiver

$$\underline{\underline{X}}_{T,n} = \underline{\underline{B}}.s(n)$$

$$E_T = \text{trace}(\underline{\underline{B}}.\underline{\underline{B}}^H) E(|s(n)|^2) = d^2 \cdot \frac{2^{n_s} - 1}{6} \cdot \text{trace}(\underline{\underline{B}}.\underline{\underline{B}}^H)$$

$$\underline{\underline{X}}_{R,n} = \underline{\underline{H}}.\underline{\underline{B}}.s(n) + \underline{\underline{W}}_n$$

The likelihood



$$\Lambda(s(n)) = - \left| \underline{\underline{X}}_{R,n} - \underline{\underline{H}}.\underline{\underline{B}}.s(n) \right|_F^2 =$$

$$= -\text{Traza} \left[\left(\underline{\underline{X}}_{R,n} - \underline{\underline{H}}.\underline{\underline{B}}.s(n) \right) \cdot \left(\underline{\underline{X}}_{R,n} - \underline{\underline{H}}.\underline{\underline{B}}.s(n) \right)^H \right] =$$

$$\rightarrow 2 \cdot \text{Re} \left[s(n)^* \cdot \text{Traza} \left(\underline{\underline{B}}^H \cdot \underline{\underline{H}}^H \cdot \underline{\underline{X}}_{R,n} \right) \right] - |s(n)|^2 \cdot \text{Traza} \left[\underline{\underline{B}}^H \cdot \underline{\underline{H}}^H \cdot \underline{\underline{H}} \cdot \underline{\underline{B}} \right]$$

ML Receiver

$$\underline{\underline{A}} = \underline{\underline{H}} \cdot \underline{\underline{B}} \quad \text{Optimum receiver}$$

when $\Lambda(s(n)) > \Lambda(s(m)) \quad \forall m \neq n$ decide $s(n)$

After some manipulations and with $\tilde{s}(n) = s(n) - s(m)$

$$\text{Traza} \left[\left(\underline{\underline{B}}^H \cdot \underline{\underline{H}}^H \cdot \underline{\underline{H}} \cdot \underline{\underline{B}} \right) \cdot |\tilde{s}(n)|^2 \right] > 2 \cdot \text{Re} \left[\tilde{s}(n) \cdot \text{Traza} \left(\underline{\underline{W}}_n^H \cdot \underline{\underline{H}} \cdot \underline{\underline{B}} \right) \right]$$

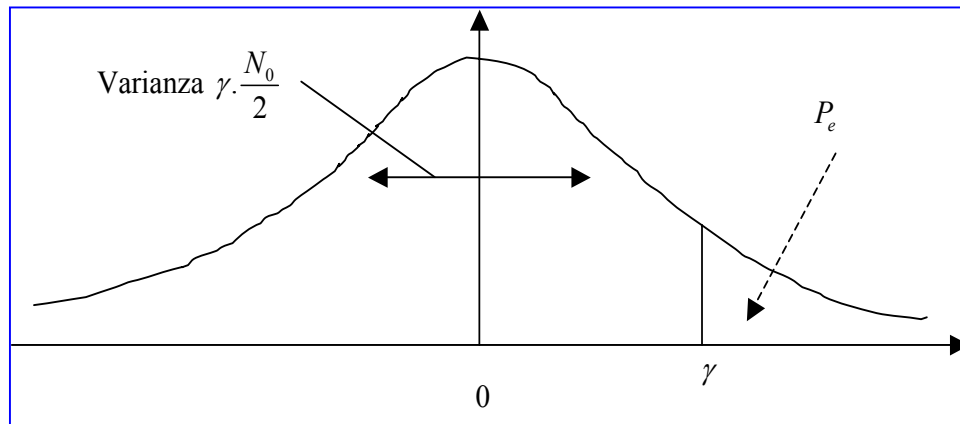
The global Tx-H matrix

define

$$\gamma = d^2 \cdot \text{Trace} \left(\underline{\underline{H}}^H \cdot \underline{\underline{R}}_0^{-1} \cdot \underline{\underline{H}} \cdot \underline{\underline{B}} \cdot \underline{\underline{B}}^H \right) = d^2 \cdot \text{Trace} \left[\underline{\underline{R}}_H \cdot \left(\underline{\underline{B}} \cdot \underline{\underline{B}}^H \right) \right]$$

The Probability of Error

$$\text{Traza} \left[\left(\underline{\underline{B}}^H \underline{\underline{H}}^H \underline{\underline{H}} \underline{\underline{B}} \right) \cdot |\tilde{s}(n)|^2 \right] > 2 \cdot \text{Re} \left[\tilde{s}(n) \cdot \text{Traza} \left(\underline{\underline{W}}_n^H \underline{\underline{H}} \underline{\underline{B}} \right) \right]$$



Error only to the nearest neighborhood

$$\Pr(s(n) \rightarrow s(m)) = P_e = Q \left(\sqrt{\frac{\gamma \cdot d^2}{2 \cdot N_0}} \right)$$

Tx Optimization

$$d^2 \text{trace}(\underline{\underline{R}}_H \cdot \underline{\underline{B}} \cdot \underline{\underline{B}}^H) \Big|_{MAX}$$
$$s.t. \quad E_s \cdot \text{trace}(\underline{\underline{B}} \cdot \underline{\underline{B}}^H) = E_T$$

Max. Quality with a given power budget

The channel matrix

$$\underline{\underline{R}}_H = \underline{\underline{H}}^H \underline{\underline{R}}_0^{-1} \underline{\underline{H}}$$

The Tx matrix and constraint

$$\underline{\underline{Q}} = (\underline{\underline{B}} \cdot \underline{\underline{B}}^H) \cdot E_s = (\underline{\underline{B}} \cdot \underline{\underline{B}}^H) \cdot d^2 \cdot \frac{2^{n_b} - 1}{6}$$

$$\text{Trace}(\underline{\underline{Q}}) = d^2 \cdot \frac{2^{n_b} - 1}{6} \cdot \text{Trace}(\underline{\underline{B}} \cdot \underline{\underline{B}}^H) \leq E_T$$

Full CSIT Design

$$d^2 \text{trace}(\underline{\underline{R}}_H \cdot \underline{\underline{B}} \cdot \underline{\underline{B}}^H) \Big|_{MAX}$$
$$s.t. \quad E_s \cdot \text{trace}(\underline{\underline{B}} \cdot \underline{\underline{B}}^H) = E_T$$

Since, for definite positive matrixes, we have:

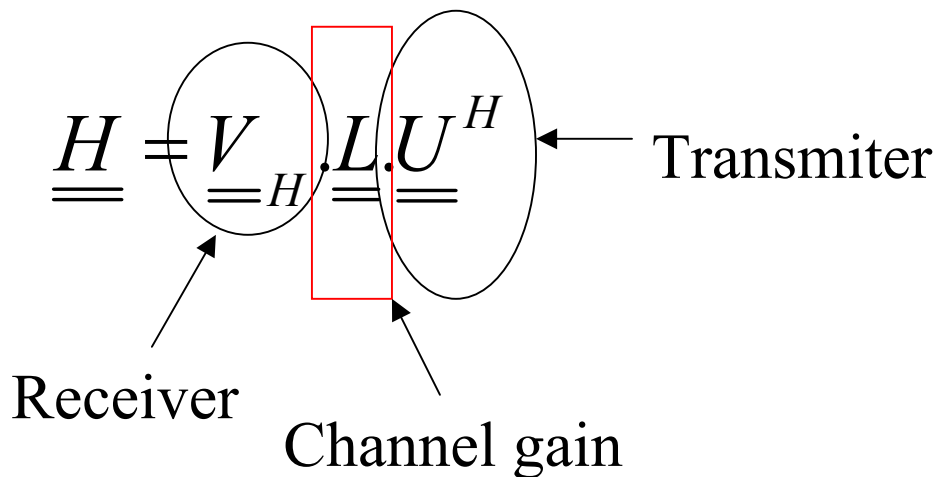
$$\text{Trace}(\underline{\underline{A}} \cdot \underline{\underline{B}}) \leq \lambda_{\max}(\underline{\underline{A}}) \cdot \text{Trace}(\underline{\underline{B}})$$

Equal when:

$$\underline{\underline{B}} = E_T \cdot \underline{\underline{e}}_{\max} \cdot \underline{\underline{cs}}^H \quad \forall \underline{\underline{cs}} \text{ unit norm}$$

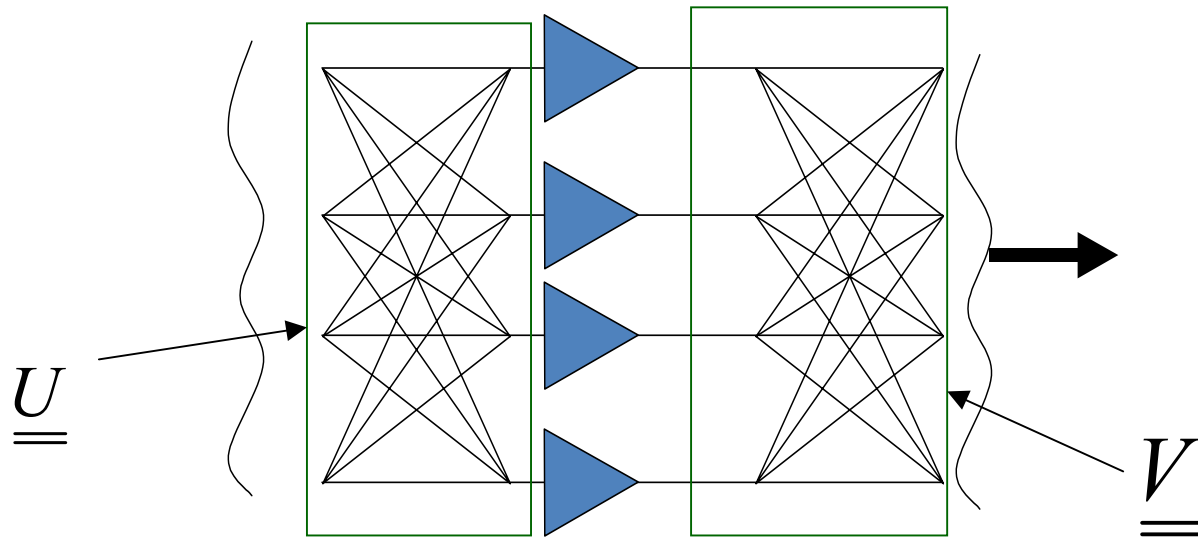
Full CSIT Architecture and Performance

$$P_e = Q\left(\sqrt{\frac{E_T}{N_0} \cdot \lambda_{MAX}(\underline{\underline{R}}_H) \cdot \frac{3}{2^{n_s} - 1}}\right)$$

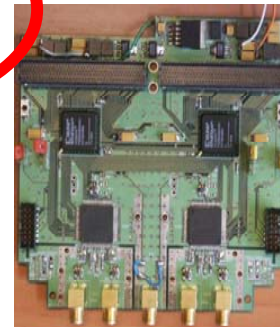
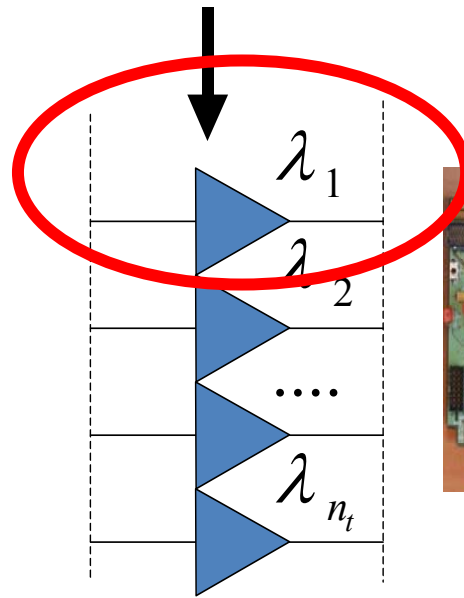


$$\underline{\underline{A}} = \underline{\underline{H}} \cdot \underline{\underline{B}} = \left(\sum_{r=1}^{\min(n_T, n_R)} \lambda_r \cdot \underline{\underline{v}}_r \cdot \underline{\underline{u}}_r^H \right) \cdot E_T \cdot \underline{\underline{u}}_{\max} = \lambda_{\max} \cdot E_T \cdot \underline{\underline{v}}_{\max}$$

Intuitive Explanation



Just use the
best channel
for all the
available Tx
power



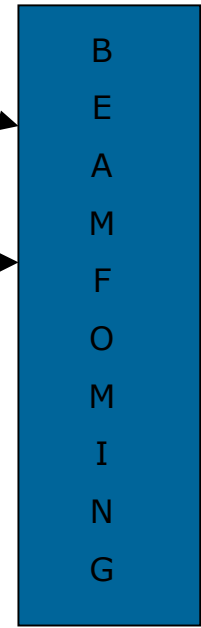


Spatial diversity



Spreading cs

Time diversity

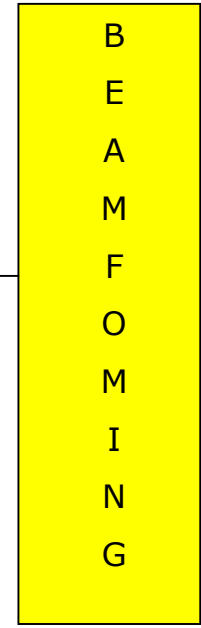


Code diversity $cs(t)$

Space diversity $Rank(H^H H)$



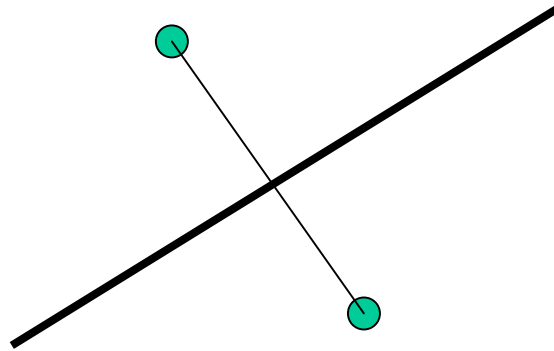
De-spreading cs



Trades (Quality, Access,..)
with capacity or "pcu"

Instantaneous Detection

$$\Lambda(s(n)) = 2 \cdot \text{Re} \left[s(n)^* \cdot \text{Traza} \left(\underline{\underline{B}}^H \cdot \underline{\underline{H}}^H \cdot \underline{\underline{X}}_{R,n} \right) \right] - |s(n)|^2 \cdot \text{Traza} \left[\underline{\underline{B}}^H \cdot \underline{\underline{H}}^H \cdot \underline{\underline{H}} \cdot \underline{\underline{B}} \right]$$



The nearest neighborhood

Constant modulus
constellation (BPSK,
QPSK, M-PSK)

$$\text{Re} \left[\text{Traza} \left(\underline{\underline{B}}^H \cdot \underline{\underline{H}}^H \cdot \underline{\underline{X}}_{R,n} \right) \right] > 0$$

$$\text{Im} \left[\text{Traza} \left(\underline{\underline{B}}^H \cdot \underline{\underline{H}}^H \cdot \underline{\underline{X}}_{R,n} \right) \right] > 0$$



The maximum likelihood detector:

$$\left(\underline{\underline{X}}_{Rn} - \underline{\underline{H}} \cdot \underline{\underline{B}} s(n) \right)^H \underline{\underline{R}}_0^{-1} \left(\underline{\underline{X}}_{Rn} - \underline{\underline{H}} \cdot \underline{\underline{B}} s(n) \right)$$

with $\underline{\underline{R}}_0^{-1} = \underline{\underline{R}}_0^{-1/2} \cdot \left(\underline{\underline{\Phi}} \cdot \underline{\underline{\Phi}}^H \right) \underline{\underline{R}}_0^{-1/2}$

$$\Lambda(s(n)) = \left| \underline{\underline{\Phi}}^H \cdot \underline{\underline{R}}_0^{-1/2} \underline{\underline{X}}_{Rn} - \underline{\underline{\Phi}}^H \cdot \underline{\underline{R}}_0^{-1/2} \underline{\underline{H}} \cdot \underline{\underline{B}} \cdot s(n) \right|_F$$

$$\underline{\underline{A}}^H$$

With the previous design:

$$\Lambda(s(n)) = \left| \underline{\underline{v}}_{\max}^H \cdot \underline{\underline{R}}_0^{-1/2} \cdot \underline{\underline{X}}_{Rn} - \lambda_{\max} \left(\underline{\underline{R}}_H \right) \cdot s(n) \right|^2$$

Enabling the symbol by symbol detection



CSIR Only (Define a Game)

Define PLAYERS

TX Engineer

Channel

Define payoffs

Minimize P_E

Destroy
Rx Energy

Define strategy of
the game

Channel plays first, Tx
engineer plays second. Only
one pass

Constrains/player

E_T

$\text{Trace}(\underline{R}_H) \geq \rho$

CSIR Only

$$\max_{\underline{\underline{Q}}} \left(\min_{\underline{\underline{R}}_h} \left(\underline{\underline{R}}_H \cdot \underline{\underline{Q}} \right) \right)$$

One play game

$$\underline{\underline{R}}_H = \underline{\underline{H}}^H \cdot \underline{\underline{R}}_0^{-1} \cdot \underline{\underline{H}} \quad \underline{\underline{Q}} = \underline{\underline{B}} \cdot \underline{\underline{B}}^H$$

$$\text{Traza} \left(\underline{\underline{R}}_H \cdot \underline{\underline{B}} \cdot \underline{\underline{B}}^H \right) \geq \lambda_{\min} \left(\underline{\underline{B}} \cdot \underline{\underline{B}}^H \right) \cdot \text{Traza} \left(\underline{\underline{R}}_H \right)$$



Channel plays

Tx Eng. plays as maximize the minimum eigenvalue when the trace is constrained

$$\underline{\underline{B}} = \left(\frac{E_T}{n_T} \right)^{1/2} \cdot \underline{\underline{\Pi}} \quad \text{with} \quad \underline{\underline{\Pi}} \cdot \underline{\underline{\Pi}}^H = \underline{\underline{I}}_{n_T}$$



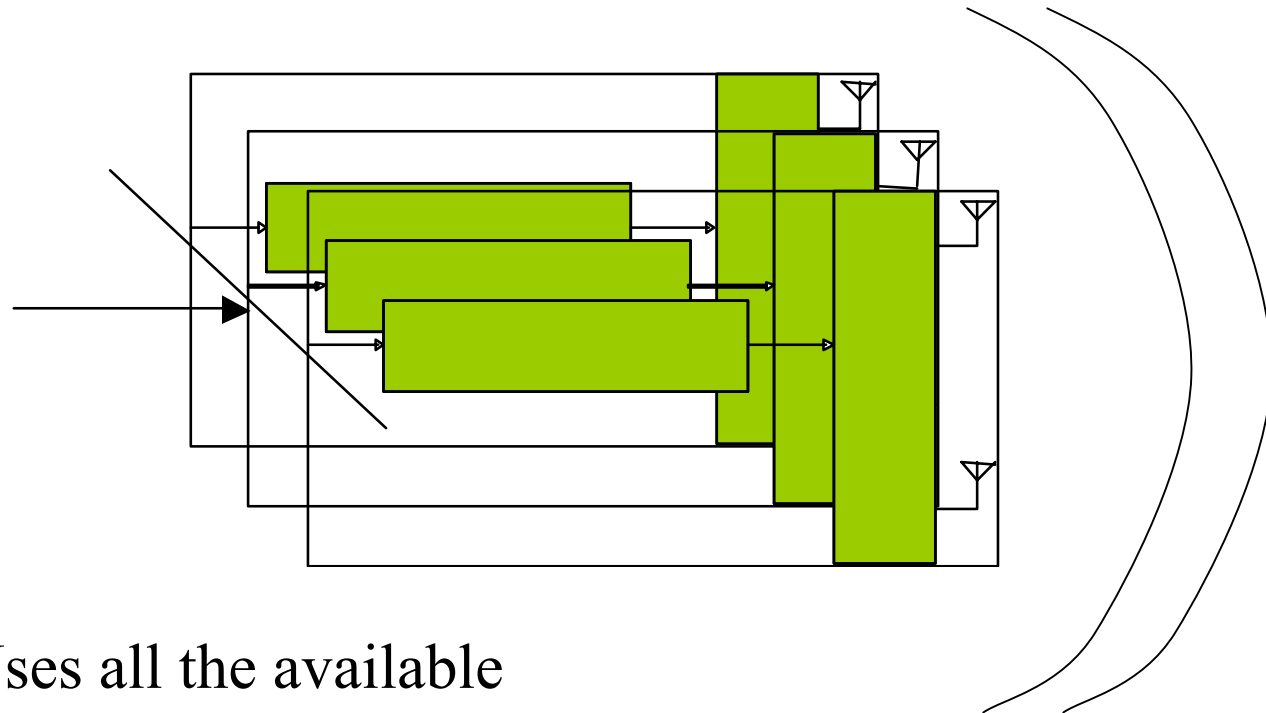
The CSIR Performance

$$\underline{\underline{B}} = \underline{\underline{\Pi}} \cdot \sqrt{\frac{k_2}{n_T}} \quad y \quad \gamma = 4E_s \cdot \text{Traza} \left(\underline{\underline{R}}_{\underline{\underline{H}}} \cdot \underline{\underline{B}} \cdot \underline{\underline{B}}^H \right) = \frac{4 \cdot E_s \cdot k_2}{n_T} \cdot \text{Traza} \left(\underline{\underline{R}}_{\underline{\underline{H}}} \right)$$

$$P_e^{NO-CSI} = Q \left(\sqrt{\frac{E_T}{N_0} \cdot \left(\frac{\text{Traza}(\underline{\underline{R}}_{\underline{\underline{H}}})}{n_T} \right) \cdot \left(\frac{3}{(2^{n_s} - 1)} \right)} \right)$$

We pass from the maximum eigenvalue gain to the arithmetic mean of the eigenvalues.

The CSIR only Strategy



Uses all the available
eigenmodes

UPA

Uniform Power
Allocation

PCU and Increase Rate

Since $\underline{\underline{B}}$ Is unitary, it has to be full rank, i.e. Size n_T by n_T (at least)

One symbol for n_T channel uses \rightarrow Rate $1/n_T$

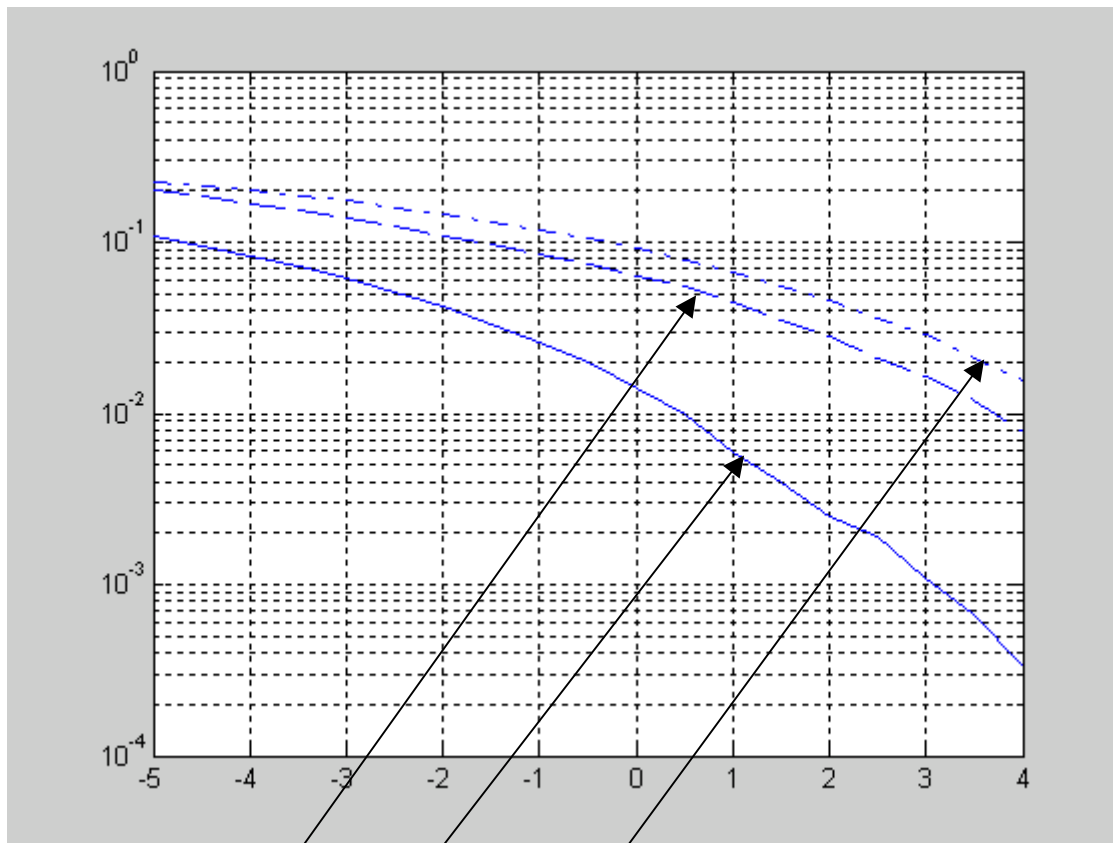
It is possible to increase this rate??

SPACE-TIME BLOCK CODES






BER



BER versus SNR for a MIMO system without CSIT. De abajo a arriba: MIMO(4,4) six streams and four PCUs ($r=3/4$); MIMO(3,3); MIMO(2,2) four streams and 2 PCUs ($r=1$).



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