

innovating communications

The Centre Tecnològic de Telecomunicacions de Catalunya

A gateway to advanced communication technologies

SPACE-TIME CODING

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CODIGOS OSTBC

The coder matrix $\underline{\underline{B}} \cdot \underline{\underline{B}}^H = \underline{\underline{I}}_{n_T}$

The Tx and Rs signals as well as the estimated symbol are:

$$\begin{aligned} \underline{\underline{X}}_{T,n} &= \underline{\underline{B}} \cdot s1(n) \\ \underline{\underline{X}}_{R,n} &= \underline{\underline{H}} \cdot \underline{\underline{X}}_{T,n} + \underline{\underline{W}}_n \\ \hat{s1} &= \text{Traza} \left[\underline{\underline{B}}^H \underline{\underline{H}}^H \cdot \underline{\underline{X}}_{R,n} \right] \end{aligned}$$

For $n_T=2$ there are several possibilities with entries entailing no operation

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \quad \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \dots$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad \dots$$

For two PAM symbols (real) and 2 antennas

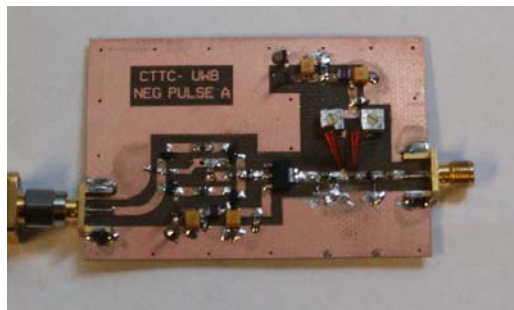
$$\underline{\underline{X}}_T = \underline{\underline{B}}_1 .s1 + \underline{\underline{B}}_2 .s2$$

$$\hat{s}1 = \underset{\text{desired}}{\text{Traza}} \left[\underline{\underline{B}}_1^H . \underline{\underline{R}}_{\underline{\underline{H}}} . \underline{\underline{B}}_1 \right] .s1 + \underset{\text{ISI}}{\text{Traza}} \left[\underline{\underline{B}}_1^H . \underline{\underline{R}}_{\underline{\underline{H}}} . \underline{\underline{B}}_2 \right] .s2$$

The no-ISI constrain is:

$$\text{Traza} \left[\underline{\underline{B}}_1^H . \underline{\underline{R}}_{\underline{\underline{H}}} . \underline{\underline{B}}_2 \right] = \text{Traza} \left[\underline{\underline{R}}_{\underline{\underline{H}}} . \underline{\underline{B}}_2 . \underline{\underline{B}}_1^H \right] = 0 \Rightarrow \underline{\underline{B}}_2 . \underline{\underline{B}}_1^H = \underline{\underline{0}}?$$

Not orthogonal just to be amicable



$$\underline{\underline{B}}_1 . \underline{\underline{B}}_2^H = - \underline{\underline{B}}_2 . \underline{\underline{B}}_1^H$$

$$\underline{\underline{B}}_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \underline{\underline{B}}_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$\underline{\underline{X}}_T = \underline{\underline{B}}_1 \cdot s1 + \underline{\underline{B}}_2 \cdot s2 = \begin{pmatrix} s1 & -s2 \\ s2 & s1 \end{pmatrix}$$

To further achieve full-rate, we need two additional matrices, that being amicable in order to detect two imaginary parts, do not promote ISI with the real symbols.

$$\underline{\underline{B}}_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \underline{\underline{B}}_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad \underline{\underline{B}}_3 \quad \underline{\underline{B}}_4$$

$$\underline{\underline{X}}_T = \underline{\underline{B}}_1 \cdot s1 + \underline{\underline{B}}_2 \cdot s2 + j \cdot \underline{\underline{B}}_3 \cdot s3 + j \cdot \underline{\underline{B}}_4 \cdot s4$$

$$\underline{\underline{B}}_1 \cdot \underline{\underline{B}}_3^H = \underline{\underline{B}}_3 \cdot \underline{\underline{B}}_1^H$$

$$\underline{\underline{B}}_2 \cdot \underline{\underline{B}}_4^H = \underline{\underline{B}}_4 \cdot \underline{\underline{B}}_2^H$$

It is easy to check that
this is the constraint---→

In summary:

$$\begin{aligned} \underline{\underline{B}}_1 \cdot \underline{\underline{B}}_1^H &= \underline{\underline{B}}_2 \cdot \underline{\underline{B}}_2^H = \underline{\underline{B}}_3 \cdot \underline{\underline{B}}_3^H = \underline{\underline{B}}_4 \cdot \underline{\underline{B}}_4^H = \underline{\underline{I}}_2 \\ \underline{\underline{B}}_1 \cdot \underline{\underline{B}}_2^H &= -\underline{\underline{B}}_2 \cdot \underline{\underline{B}}_1^H & \underline{\underline{B}}_4 \cdot \underline{\underline{B}}_3^H &= -\underline{\underline{B}}_3 \cdot \underline{\underline{B}}_4^H \\ \underline{\underline{B}}_1 \cdot \underline{\underline{B}}_3^H &= \underline{\underline{B}}_3 \cdot \underline{\underline{B}}_1^H & \underline{\underline{B}}_2 \cdot \underline{\underline{B}}_4^H &= \underline{\underline{B}}_4 \cdot \underline{\underline{B}}_2^H \end{aligned}$$

The Alamouti's code:

$$\underline{\underline{B}}_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \underline{\underline{B}}_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad \underline{\underline{B}}_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \underline{\underline{B}}_4 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\underline{\underline{X}}_T = \underline{\underline{B}}_1 \cdot s1 + \underline{\underline{B}}_2 \cdot s2 + \underline{\underline{B}}_3 \cdot j \cdot s3 + \underline{\underline{B}}_4 \cdot j \cdot s4 = \begin{pmatrix} s1 + j \cdot s3 & -s2 + j \cdot s4 \\ s2 + j \cdot s4 & s1 - j \cdot s3 \end{pmatrix} = \begin{pmatrix} z1 & -z2^* \\ z2 & z1^* \end{pmatrix}$$

The receiver----->

$$\hat{s}1 = \text{Re} \left(\text{Traza} \left[\underline{\underline{B}}_1^H \cdot \underline{\underline{R}}_H \cdot \underline{\underline{X}}_R \right] \right)$$

$$\hat{s}2 = \text{Re} \left(\text{Traza} \left[\underline{\underline{B}}_2^H \cdot \underline{\underline{R}}_H \cdot \underline{\underline{X}}_R \right] \right)$$

$$\hat{s}3 = \text{Im} \left(\text{Traza} \left[\underline{\underline{B}}_1^H \cdot \underline{\underline{R}}_H \cdot \underline{\underline{X}}_R \right] \right)$$

$$\hat{s}4 = \text{Im} \left(\text{Traza} \left[\underline{\underline{B}}_2^H \cdot \underline{\underline{R}}_H \cdot \underline{\underline{X}}_R \right] \right)$$



Unfortunately no such full-rate codes exist for any number of antennas. There are solution for rates lower than one like the code shown below for 4 antennas and rate $\frac{3}{4}$.

$$\begin{bmatrix} s1 & 0 & s2 & -s3 \\ 0 & s1 & s3^* & s2^* \\ -s2^* & -s3 & s1^* & 0 \\ s3^* & -s2 & 0 & s1^* \end{bmatrix}$$



Convolutional S-T Codes: Trellis codes

output input state

$$\underline{x} = \left[\underline{G}_1 \cdot \underline{a} + \underline{G}_2 \cdot \underline{b} \right]^{\square}$$

Measurement equation

$$\underline{b} = \left[\underline{G}_3 \cdot \underline{a} + \underline{G}_4 \cdot \underline{b} \right]^{\oplus}$$

State equation

$\underline{a} = [a(1), a(2), \dots, a(R)]^T$ input bit

L bits per component output \underline{x}

L=2 bits Components

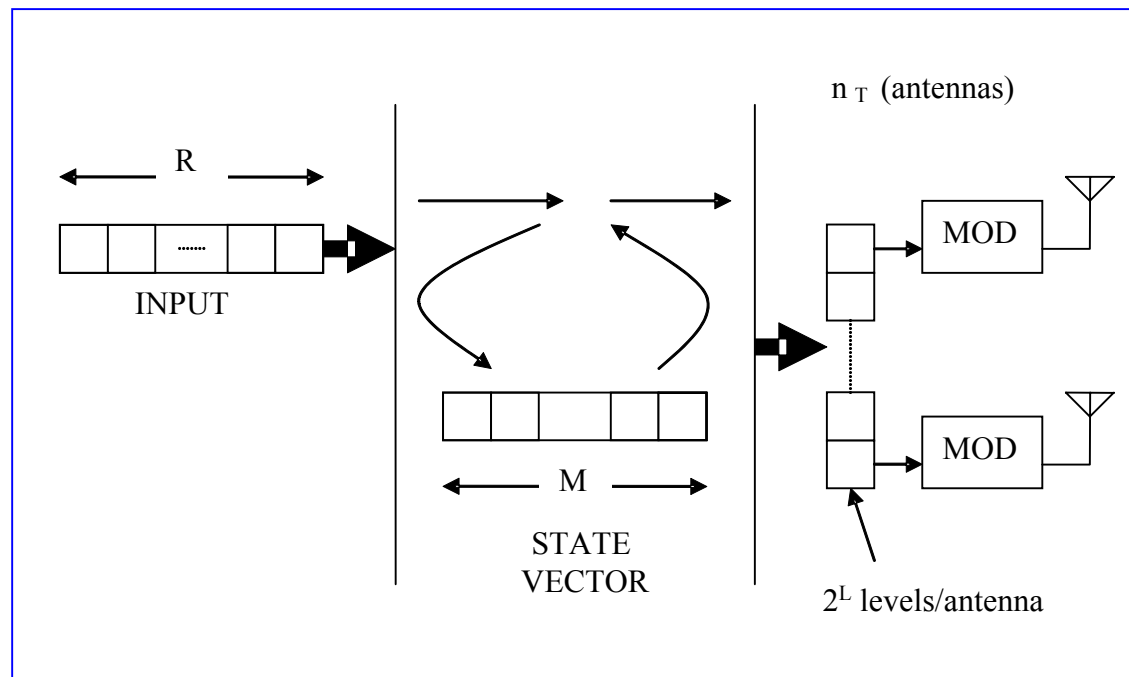
0(00), 1(01), 2(10), 3(11) that correspond to the four signals in a QPSK constellation

Code rate = R/L over n_T antennas

K bits for M components of the state vector \underline{b}

Code specification:

- Number of Tx antennas n_T
- Bits/Hz or size of the radiated constellation L
- Complexity at Tx or number of states $2^{K.M}$
- Code Rate equal to R/L



A more compact formulation of the state model is:

Grouping input and state vectors in a single one

$$\underline{c} = [a(1) \quad \dots \quad a(R) \quad b(1) \quad \dots \quad b(M)]$$

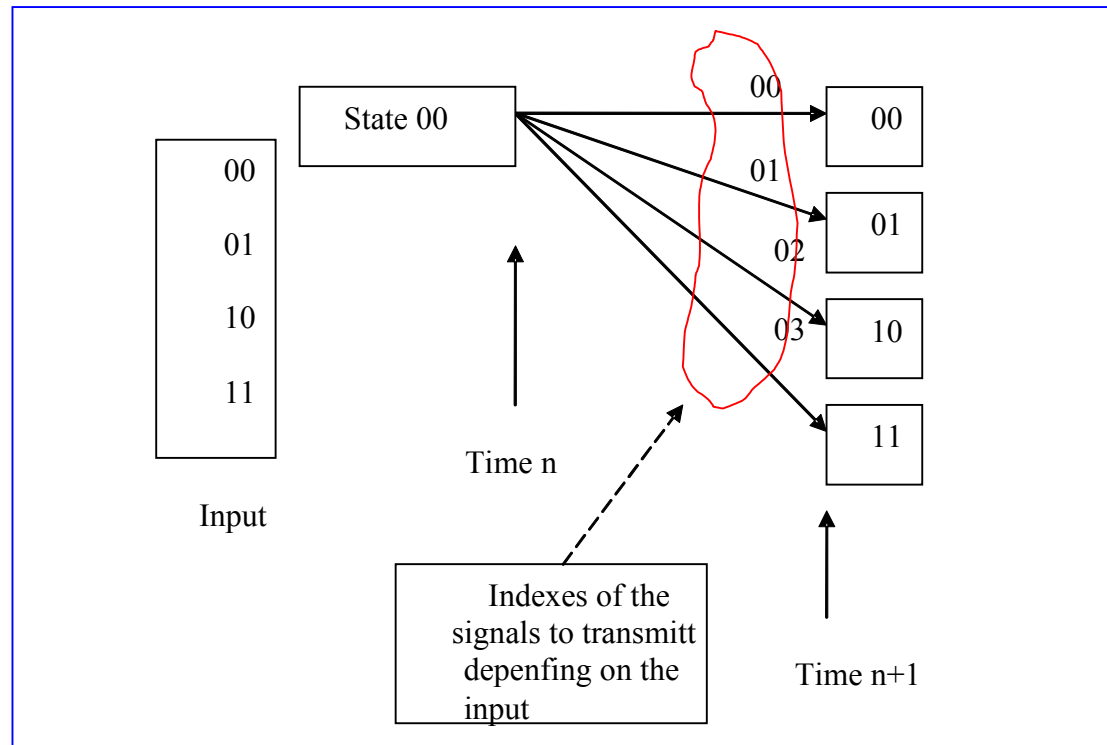
$$\underline{x} = \left[\underline{\underline{G}} \cdot \underline{c} \right]^{\oplus}$$

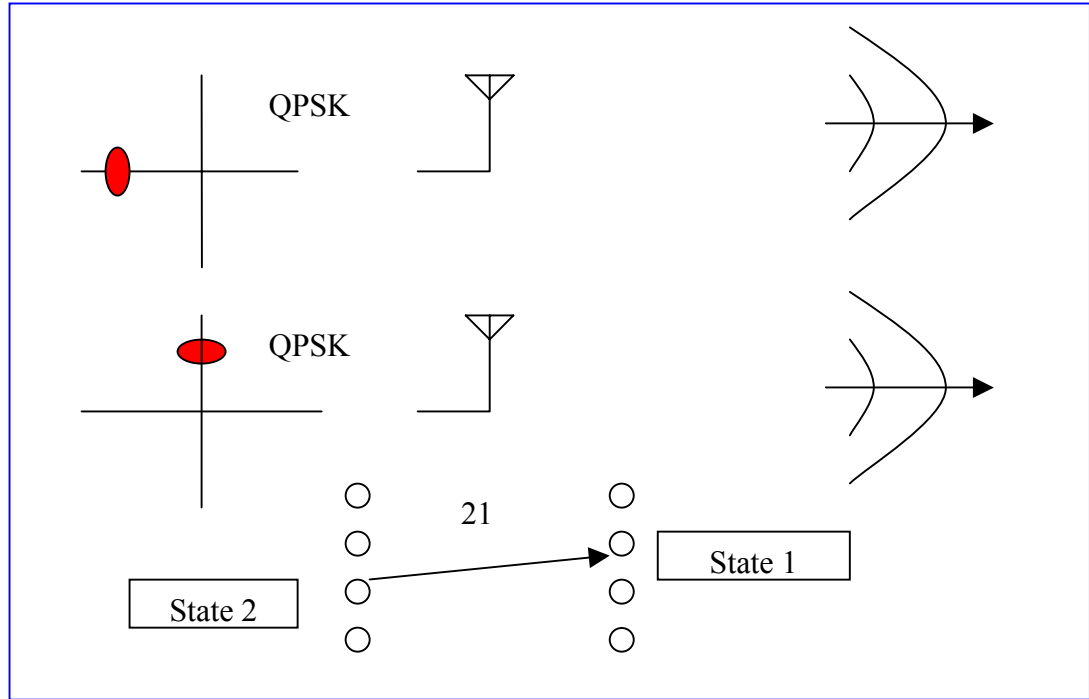
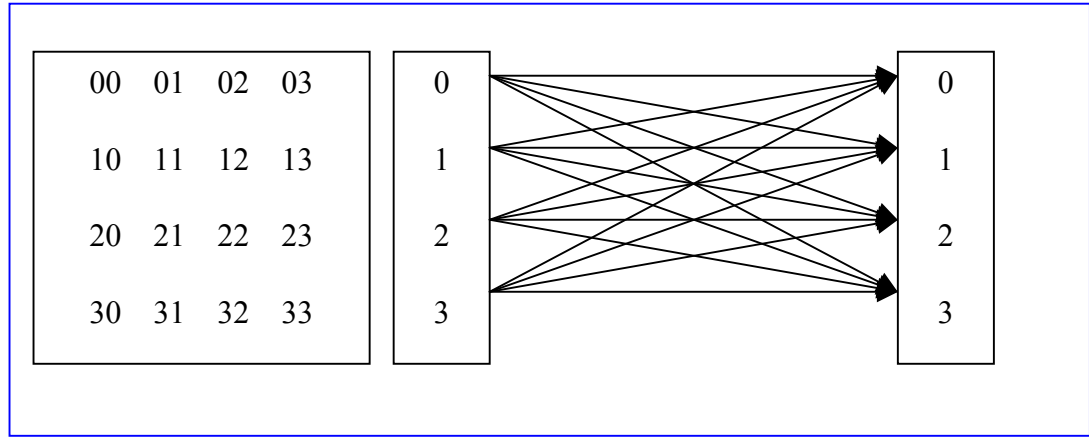
And, the state vector is formed by the last M bits after a shift of a given number of components vector \underline{c} from left to right

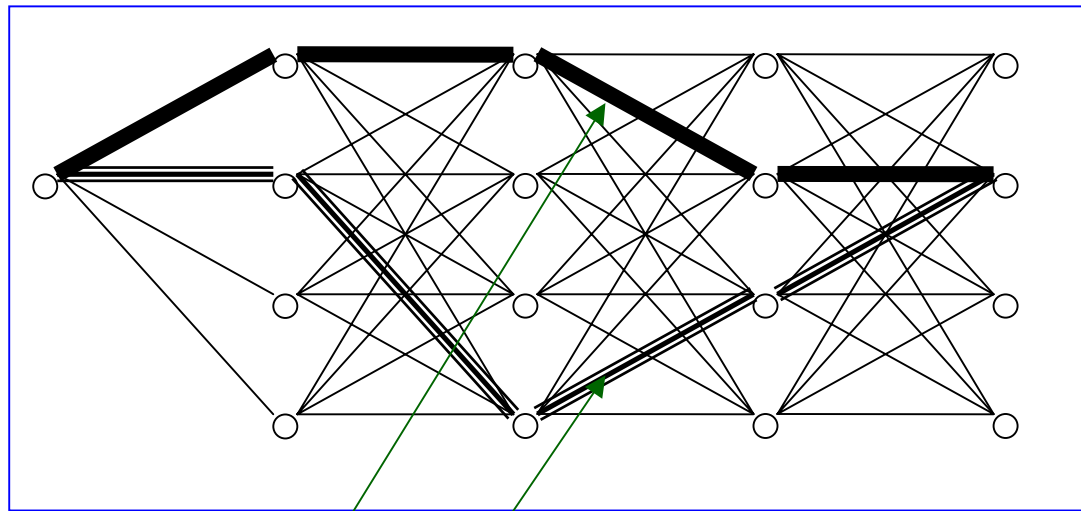
Code *st2bh2est4rate1*:

$$[a(1) \ a(2) \ b(1) \ b(2)] \cdot \begin{bmatrix} 0 & 2 \\ 0 & 1 \\ 2 & 0 \\ 1 & 0 \end{bmatrix} = [x1 \ x2]$$

Modul 2
operation for
x1 and x2







For uniform code any two pair is good. Use the $(0,0,0,0,0\dots)$ as reference.

Search for the lowest distance covered to recover the zero path (This will be the worst matrix A to be used in the BER upper bound)

$$\Pr(\underline{s}_n \Rightarrow \underline{b}_n; n = 1, N) = Q \left(\sqrt{\left(\frac{E_s}{2.N_0} \right) \cdot \text{Traza}(\underline{R}_H \cdot \underline{A})} \right)$$

For length of N channels access

$$\underline{\underline{A}} = \sum_{n=1}^N (\underline{s}_n - \underline{b}_n) \cdot (\underline{s}_n - \underline{b}_n)^H$$

And the average BER

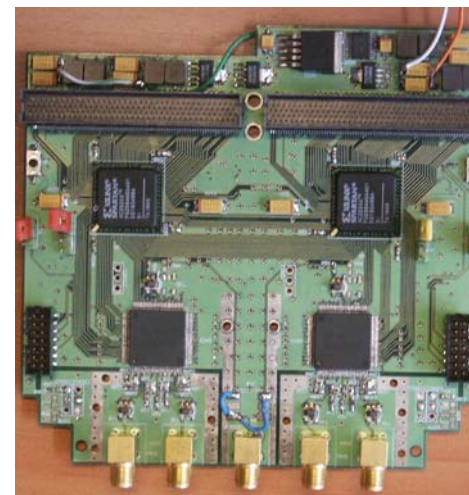
$$\Pr(\underline{I}_0 \Rightarrow \underline{I}_e) \approx k_1 \cdot \prod_{p=1}^{n_R} \frac{1}{\det \left[\underline{I}_{=n_T} + \frac{2E_S}{N_0} \cdot \underline{\underline{A}} \cdot \underline{\underline{\Sigma}}_p \right]} \approx \left| \begin{array}{l} \text{Para relaciones} \\ \text{señal a ruido} \\ \text{elevadas} \end{array} \right| \approx$$

$$\approx k_1 \cdot \left(\frac{2E_S}{N_0} \right)^{-n_R \cdot n_T} \cdot \prod_{p=1}^{n_R} \frac{1}{\det \left[\underline{\underline{A}} \cdot \underline{\underline{\Sigma}}_p \right]}$$

or

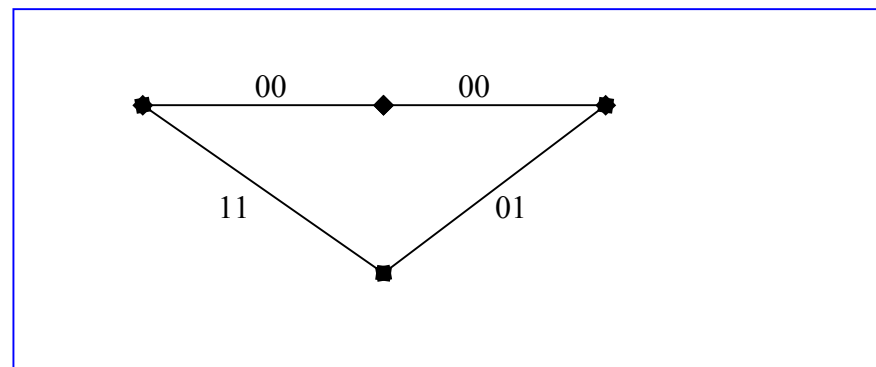
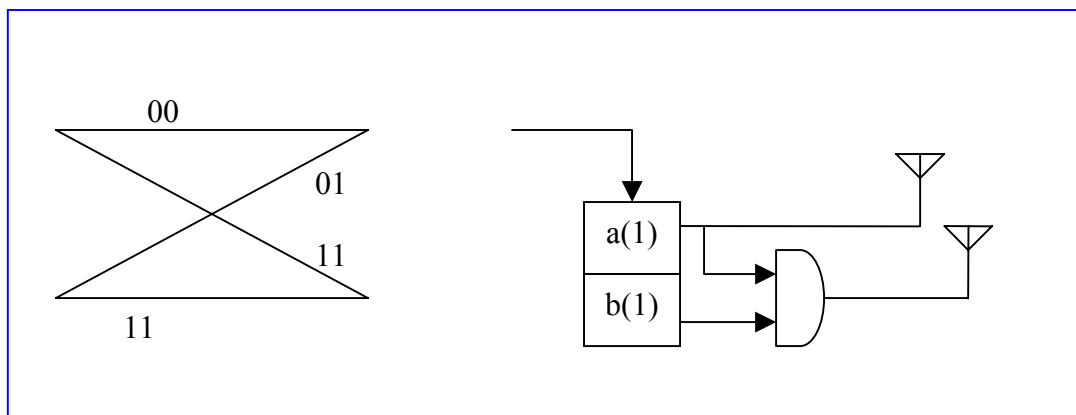
$$\Pr(\underline{I}_0 \Rightarrow \underline{I}_e) \approx k_1 \cdot \left(\frac{2E_S}{N_0} \right)^{-n_R \cdot n_T} \cdot \prod_{p=1}^{n_R} \frac{1}{\det \left[\underline{\underline{A}} \cdot \underline{\underline{\Sigma}}_p \right]} \approx k_2 \cdot \left(\frac{2E_S}{N_0} \right)^{-n_R \cdot n_T} \cdot \left(\det(\underline{\underline{A}}) \right)^{-r}$$

$$Ganancia = \left[\det(\underline{\underline{A}})_{mayor} \right]^{-r}$$



For $st2bh1est2rate0.5$

$$\underline{\underline{G}} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$



$$\underline{s}_1 - \underline{b}_1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \quad y \quad \underline{s}_2 - \underline{b}_2 = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$\underline{\underline{A}} = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \cdot [2 \quad 2] + \begin{bmatrix} 0 \\ 2 \end{bmatrix} \cdot [0 \quad 2] = \begin{pmatrix} 4 & 4 \\ 4 & 4 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 4 & 4 \\ 4 & 8 \end{pmatrix}$$

$$\det(\underline{\underline{A}}) = 16 \quad \text{Ganancia} = \sqrt{16} = 4$$



St2bh2est8	Gain 2	$\underline{\underline{G}}^T = \begin{bmatrix} 0 & 2 & 1 & 0 & 2 \\ 2 & 1 & 0 & 2 & 2 \end{bmatrix}$
St2bh2est16	Gain $\sqrt{32}$	$\underline{\underline{G}}^T = \begin{bmatrix} 0 & 2 & 1 & 1 & 2 & 0 \\ 2 & 2 & 1 & 2 & 0 & 2 \end{bmatrix}$
St2bh1est2	Gain 4	$\underline{\underline{G}}^T = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$
St2bh1est4	Gain $\sqrt{48}$	$\underline{\underline{G}}^T = \begin{bmatrix} 0 & 2 & 1 & 0 & 2 \\ 2 & 1 & 0 & 2 & 2 \end{bmatrix}$
St2bh1est8	Gain $\sqrt{80}$	$\underline{\underline{G}}^T = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$
St2bh1est16	Gain $\sqrt{128}$	$\underline{\underline{G}}^T = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$
St3bh1est8	Gain $\sqrt[3]{256}$	$\underline{\underline{G}}^T = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$





Codes for No-CSI at Rx

Assume that $\log_2 M$ bits have to be transmitted using N access to the channel with n_T antennas. We will use M matrixes of n_T by N

$$\underline{\underline{C}}_m \quad m = 1, M$$

UPA at Tx

$$\underline{\underline{C}}_m \cdot \underline{\underline{C}}_m^H = \underline{\underline{I}}_{n_T} \quad \forall m = 1, M$$

$$\text{At Rx} \rightarrow \underline{\underline{Y}}_R = \left(\frac{2E_s}{N_0} \right)^{1/2} \underline{\underline{H}} \cdot \underline{\underline{C}}_0 + \underline{\underline{W}} = \rho^{1/2} \underline{\underline{H}} \cdot \underline{\underline{C}}_0 + \underline{\underline{W}}$$

$$\underline{\underline{\Sigma}} = E \left[\underline{\underline{Y}}_R^H \cdot \underline{\underline{Y}}_R \right] = \underline{\underline{I}} + \underline{\underline{C}}_0^H \cdot E \left[\underline{\underline{H}}^H \cdot \underline{\underline{H}} \right] \cdot \underline{\underline{C}}_0 = \underline{\underline{I}} + \rho \cdot \underline{\underline{C}}_0^H \cdot \underline{\underline{R}}_{HA} \cdot \underline{\underline{C}}_0 = \underline{\underline{I}} + \rho |H_0|^2 \cdot \underline{\underline{C}}_0^H \cdot \underline{\underline{C}}_0$$

The ML receiver is:

$$\begin{aligned} \Pr \left(\underline{\underline{Y}}_R / \underline{\underline{C}}_0 \right) &= k_0 \cdot \exp - \left[\text{Traza} \left(\underline{\underline{Y}}_R \cdot \underline{\underline{\Sigma}}^{-1} \cdot \underline{\underline{Y}}_R^H \right) \right] = \\ &= k_0 \cdot \exp - \left[\text{Traza} \left(\underline{\underline{Y}}_R \cdot \left(\underline{\underline{I}} - \rho \cdot \underline{\underline{C}}_0^H \cdot \underline{\underline{C}}_0 \right) \cdot \underline{\underline{Y}}_R^H \right) \right] \end{aligned}$$

$$\hat{m} = \arg \max_{\underline{\underline{C}}_m, m=1, M} \left[\text{Traza} \left(\underline{\underline{Y}}_R \cdot \underline{\underline{C}}_m^H \cdot \underline{\underline{C}}_m \cdot \underline{\underline{Y}}_R^H \right) \right]$$

Perfect detection occurs when:

$$\text{Traza} \left(\underline{\underline{Y}}_R \cdot \underline{\underline{C}}_0^H \cdot \underline{\underline{C}}_0 \cdot \underline{\underline{Y}}_R^H \right) > \text{Traza} \left(\underline{\underline{Y}}_R \cdot \underline{\underline{C}}_1^H \cdot \underline{\underline{C}}_1 \cdot \underline{\underline{Y}}_R^H \right)$$

$$\rho \cdot \text{Traza} \left[\underline{\underline{H}} \cdot \left(\underline{\underline{I}} - \underline{\underline{C}}_0 \cdot \underline{\underline{C}}_1^H \cdot \underline{\underline{C}}_1 \cdot \underline{\underline{C}}_0^H \right) \cdot \underline{\underline{H}}^H \right] > 2\rho^{1/2} \cdot \text{Re} \left\{ \text{Traza} \left[\underline{\underline{W}} \cdot \left(\underline{\underline{C}}_0^H \cdot \underline{\underline{C}}_0 - \underline{\underline{C}}_1^H \cdot \underline{\underline{C}}_1 \right) \cdot \underline{\underline{C}}_0^H \cdot \underline{\underline{H}}^H \right] \right\}$$

$$\Pr(\underline{\underline{C}}_0 \rightarrow \underline{\underline{C}}_1) \approx Q \left(\sqrt{\left(\frac{2E_s}{N_0} \right) \cdot \text{Traza} \left(\underline{\underline{H}} \cdot \underline{\underline{A}}_{\text{NOCSI}} \cdot \underline{\underline{H}}^H \right)} \right) \approx$$

$$\approx k_1 \cdot \exp \left[- \left(\left(\frac{2E_s}{N_0} \right) \cdot \text{Traza} \left(\underline{\underline{H}} \cdot \underline{\underline{A}}_{\text{NOCSI}} \cdot \underline{\underline{H}}^H \right) \right) \right]$$

$$\Pr^{AVE} \approx k_2 \cdot \prod_{p=1}^{n_R} \frac{1}{\det \left[\underline{\underline{I}} - \left(\frac{E_s}{2 \cdot N_0} \right) \underline{\underline{A}}_{\text{NOCSI}} \cdot \underline{\underline{\Sigma}}_p \right]}$$

It is important to remark the loss due to the absence of CSI at Rx

$$E_s = \left(\frac{N}{n_T} \right) \cdot E_T$$

$$\underline{\underline{A}}_{\text{CSI}} = \left(\underline{\underline{C}}_0 - \underline{\underline{C}}_1 \right) \left(\underline{\underline{C}}_0 - \underline{\underline{C}}_1 \right)^H$$

$$\underline{\underline{A}}_{\text{NOCSI}} = \underline{\underline{I}} - \underline{\underline{C}}_0 \cdot \underline{\underline{C}}_1^H \cdot \underline{\underline{C}}_1 \cdot \underline{\underline{C}}_0^H$$

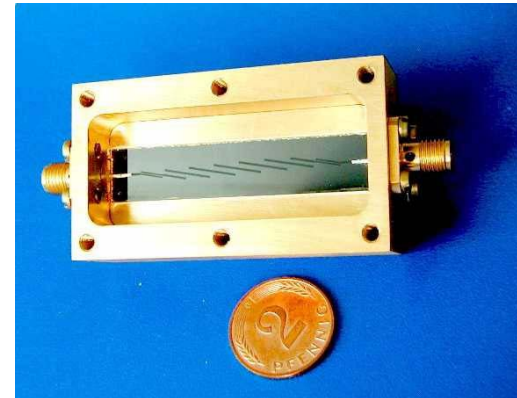
In order to compare both cases, note that:

$$\left| \underline{I} - \underline{C}_{\underline{0}} \cdot \underline{C}_{\underline{1}}^H \right|^2 = \underline{I} + \underline{C}_{\underline{0}} \cdot \underline{C}_{\underline{1}}^H \underline{C}_{\underline{1}} \cdot \underline{C}_{\underline{0}}^H - \underline{C}_{\underline{0}} \cdot \underline{C}_{\underline{1}}^H - \underline{C}_{\underline{1}} \cdot \underline{C}_{\underline{0}}^H =$$

$$2 \cdot \underline{I} - \underline{C}_{\underline{0}} \cdot \underline{C}_{\underline{1}}^H - \underline{C}_{\underline{1}} \cdot \underline{C}_{\underline{0}}^H - \underline{A}_{NOCSI} = \underline{A}_{CSI} - \underline{A}_{NOCSI}$$

Asi pues,

$$\underline{A}_{CSI} = \underline{A}_{NOCSI} + \left| \underline{I} - \underline{C}_{\underline{0}} \cdot \underline{C}_{\underline{1}}^H \right|^2$$



Codigos ST Diferenciales

Assuming that there is CSI at Rx

$$\left| \underline{X}_R - E_s^{1/2} \cdot \underline{H} \cdot \underline{C}_m \right|_F \Rightarrow \hat{\underline{C}} = \underset{\underline{C}_m; m=1, M}{\text{Max}} \left[\text{Re} \left(\text{Traza} \left(\underline{H} \cdot \underline{C}_m \cdot \underline{X}_R \right) \right) \right]$$

The probability of error is:

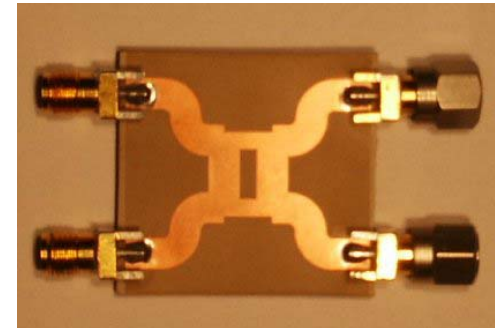
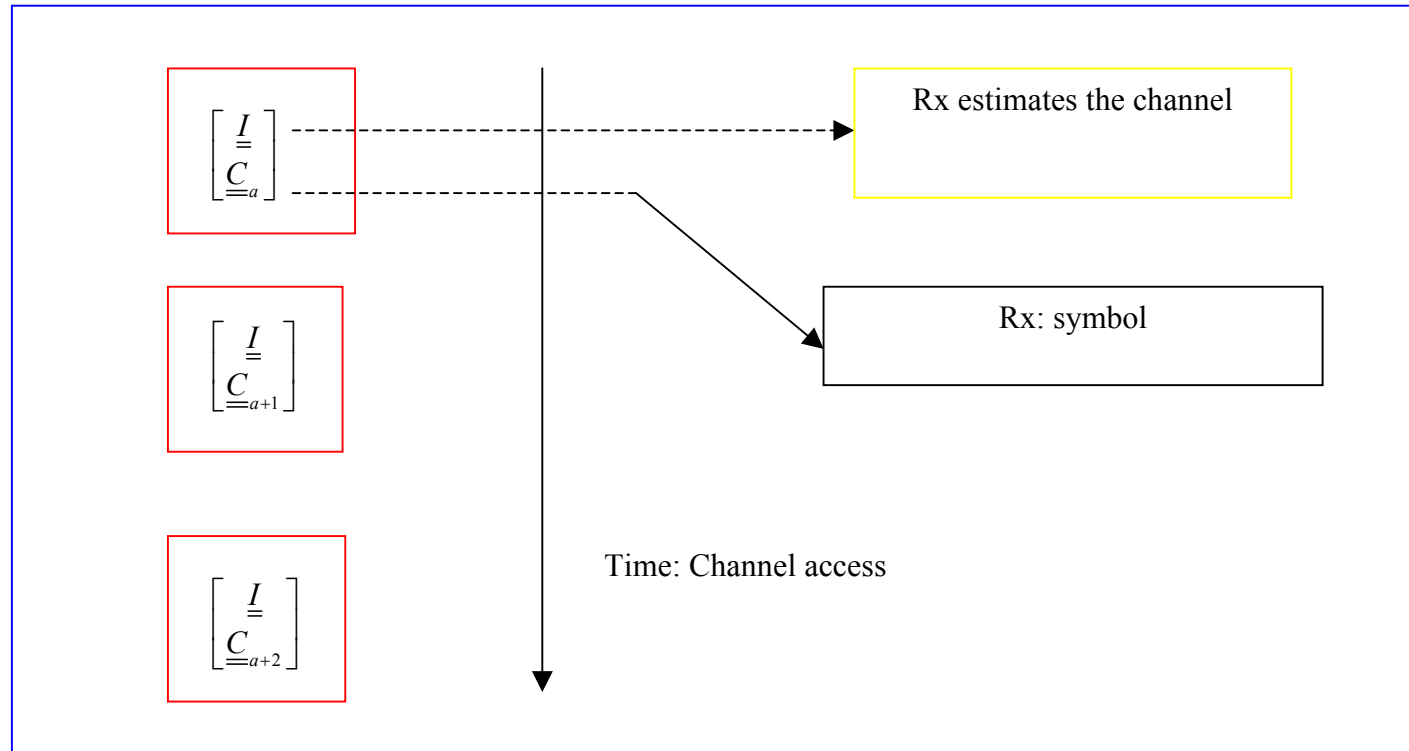
$$Pe = \Pr \left(\underline{C}_m \rightarrow \underline{C}_n; \tilde{\underline{C}} \equiv \underline{C}_m - \underline{C}_n \right) = k_0 \cdot Q \left(\sqrt{\frac{E_s}{2 \cdot N_0} \cdot \text{Traza} \left(\underline{R}_H \cdot \tilde{\underline{C}} \cdot \tilde{\underline{C}}^H \right)} \right)$$

$$Pe \approx k_1 \cdot \exp \left[-\frac{E_s}{4 \cdot N_0} \cdot \text{Traza} \left(\underline{R}_H \cdot \tilde{\underline{C}} \cdot \tilde{\underline{C}}^H \right) \right] = k_1 \cdot \exp \left[-\frac{E_s}{4 \cdot N_0} \cdot \sum_{p=1}^{n_R} h_p^H \cdot \tilde{\underline{C}} \cdot \tilde{\underline{C}}^H \cdot h_p \right]$$

$$Pe^{aver} = k_2 \cdot \prod_{p=1}^{n_R} \frac{1}{\det \left[\underline{I} + \frac{E_s}{4 \cdot N_0} \cdot \tilde{\underline{C}} \cdot \tilde{\underline{C}}^H \cdot \underline{\Sigma}_j \right]}$$

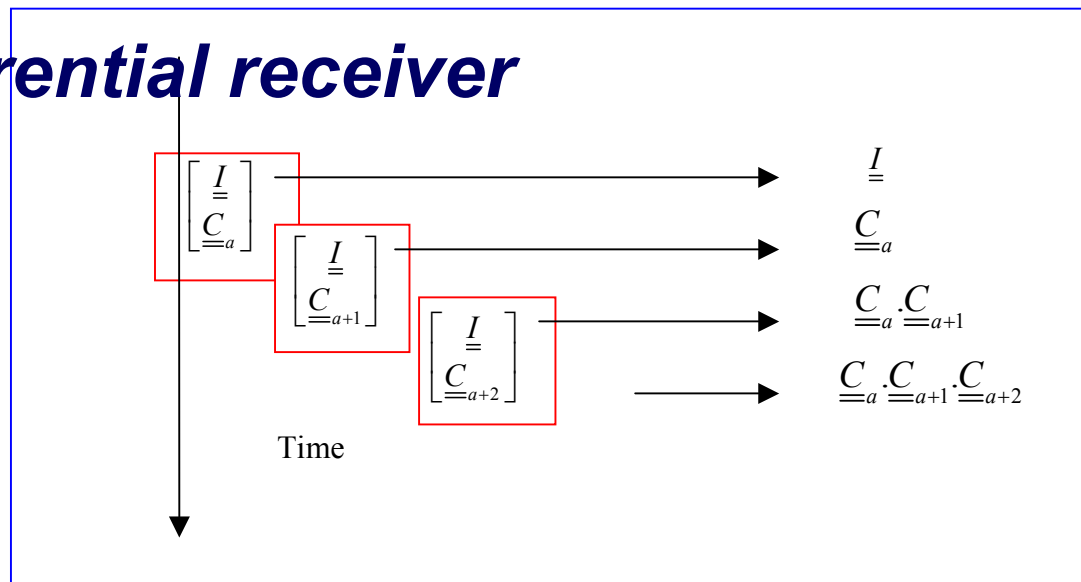


Using two symbols first estimate the channel second to decode.





The differential receiver

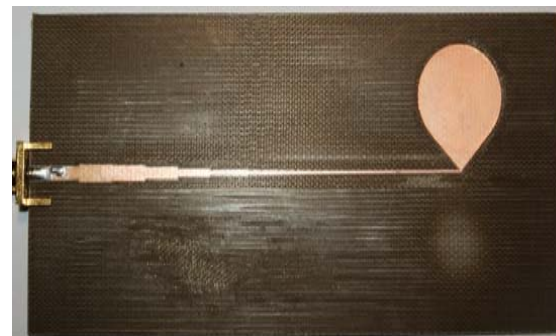


The desired word

received is
$$\underline{Z}_{\underline{n-1}} = \prod_{a=1}^{\underline{n-1}} \underline{C}_{\underline{n-a}}$$

And the received
snapshot

$$\underline{X}_{\underline{R}, \underline{n-1}} = \underline{H} \cdot \underline{Z}_{\underline{n-1}} + \underline{W}_{\underline{n-1}}$$





Hereafter it is shown that the ST code produces, in fact, a new channel together with 3 dB. increase of noise.

$$\underline{X}_{R,n} = \underline{H} \cdot \underline{Z}_{n-1} \cdot \underline{C}_{n-1} + \underline{W}_n = \left(\underline{X}_{R,n-1} - \underline{W}_{n-1} \right) \cdot \underline{C}_{n-1} + \underline{W}_n$$

$$\underline{X}_{R,n} = \underline{X}_{R,n-1} \cdot \underline{C}_{n-1} + \left(\underline{W}_n - \underline{W}_{n-1} \cdot \underline{C}_{n-1} \right) = \underline{H}_{nuevo} \cdot \underline{C}_{n-1} + \underline{W}_{nuevo}$$

Regardless the system is full-rate the decoder requires of two received codewords

$$\left| \underline{X}_{R,n} - \underline{X}_{R,n-1} \cdot \underline{C}_{n-1} \right|_F \Rightarrow \hat{\underline{C}}_{n-1} = \underset{\underline{C}_{n-1}; n=1, M}{Max} \left[\text{Re} \left(\text{Traza} \left(\underline{X}_{R,n-1} \cdot \underline{C}_{n-1} \cdot \underline{X}_{R,n}^H \right) \right) \right]$$





A compact formulation for two channel access:

Two data received $\begin{bmatrix} \underline{\underline{X}}_{R,k-1} & \underline{\underline{X}}_{R,k} \end{bmatrix}$

New codeword $\begin{bmatrix} \underline{\underline{Z}}_{k-1} & \underline{\underline{Z}}_{k-1} \cdot \underline{\underline{C}}_k \end{bmatrix} \equiv \underline{\underline{C}}_k$

with

$$\underline{\underline{C}}_k^H \underline{\underline{C}}_k = \begin{pmatrix} \underline{\underline{I}} & \underline{\underline{C}}_k \\ \underline{\underline{C}}_k^H & \underline{\underline{I}} \end{pmatrix} \quad y \quad \underline{\underline{C}}_k \underline{\underline{C}}_k^H = 2 \cdot \underline{\underline{I}}$$

Optimum detector for no-CSI at Rx that arrives to the same result

$$\text{Traza} \left[\begin{pmatrix} \underline{\underline{X}}_{R,k-1} & \underline{\underline{X}}_{R,k} \end{pmatrix} \begin{pmatrix} \underline{\underline{I}} & \underline{\underline{C}}_k \\ \underline{\underline{C}}_k^H & \underline{\underline{I}} \end{pmatrix} \begin{pmatrix} \underline{\underline{X}}_{R,k-1}^H \\ \underline{\underline{X}}_{R,k}^H \end{pmatrix} \right] = \text{Traza} \left[\underline{\underline{X}}_{R,k} \underline{\underline{C}}_k^H \underline{\underline{X}}_{R,k-1}^H \right]$$





Also, the matrix \underline{A} for the average BER is as follows:

$$\begin{aligned}
 & 2 \cdot \underline{I} - \frac{1}{2} \cdot \underline{\underline{C}}_{k0} \cdot \underline{\underline{C}}_{k1}^H \cdot \underline{\underline{C}}_{k1} \cdot \underline{\underline{C}}_{k0}^H = \\
 & = 2 \underline{I} - \frac{1}{2} \cdot \left(\underline{I} + \underline{\underline{Z}}_{k-1} \cdot \underline{\underline{C}}_{\underline{0}} \cdot \underline{\underline{C}}_{\underline{1}}^H \cdot \underline{\underline{Z}}_{k-1}^H + \underline{\underline{Z}}_{k-1} \cdot \underline{\underline{C}}_{\underline{1}} \cdot \underline{\underline{C}}_{\underline{0}}^H \cdot \underline{\underline{Z}}_{k-1}^H + \underline{I} \right) = \\
 & = \underline{I} - \frac{1}{2} \cdot \left(\underline{\underline{Z}}_{k-1} \cdot \underline{\underline{C}}_{\underline{0}} \cdot \underline{\underline{C}}_{\underline{1}}^H \cdot \underline{\underline{Z}}_{k-1}^H + \underline{\underline{Z}}_{k-1} \cdot \underline{\underline{C}}_{\underline{1}} \cdot \underline{\underline{C}}_{\underline{0}}^H \cdot \underline{\underline{Z}}_{k-1}^H \right) = \\
 & = \frac{1}{2} \left[\left(\underline{\underline{Z}}_{k-1} \cdot \underline{\underline{C}}_{\underline{0}} - \underline{\underline{Z}}_{k-1} \cdot \underline{\underline{C}}_{\underline{1}} \right) \cdot \left(\underline{\underline{Z}}_{k-1} \cdot \underline{\underline{C}}_{\underline{0}} - \underline{\underline{Z}}_{k-1} \cdot \underline{\underline{C}}_{\underline{1}} \right)^H \right] = \\
 & = \frac{1}{2} \cdot \left[\underline{\underline{Z}}_{k-1} \cdot \left(\underline{\underline{C}}_{\underline{0}} - \underline{\underline{C}}_{\underline{1}} \right) \cdot \left(\underline{\underline{C}}_{\underline{0}} - \underline{\underline{C}}_{\underline{1}} \right)^H \cdot \underline{\underline{Z}}_{k-1}^H \right] = \underline{\underline{A}}_{DIF}
 \end{aligned}$$

Where, taking into account the orthogonal character of the received codewords and the commutative property of the determinant, results identical to the CSI at Rx case with 3dB loss.



Some examples of differential ST codes

For 1 bit rate only two matrixes $\Phi = \pm \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

With initial
matrix $\underline{\underline{D}} = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

BPSK as contellation and rate 0.25 and 2
antennas, Gain 8

2 bits, rate=1, 2 antennas, rate 0.5 \rightarrow four
matrixes, BPSK, Gain 4

$$\Phi = \left\{ \pm \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \pm \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \right\}$$



QPSK, 2bits/seg/Hz, 8 codewords, Code gain 4,
Rate 1

$$\Phi = \left\{ \pm \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \pm \begin{pmatrix} j & 0 \\ 0 & -j \end{pmatrix}, \pm \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \pm \begin{pmatrix} 0 & j \\ j & 0 \end{pmatrix} \right\}$$

“quaternion” similar to Alamouti’s code


For higher rates the codewords are formed as:

$$w_Q = \exp(j2\pi / Q)$$

$$\Phi = \left\{ \begin{pmatrix} 0 & w_Q \\ 1 & 0 \end{pmatrix}^m ; m = 0, 2Q-1 \right\}$$

Q=8 Rate 2 Gain 1.531 // Q=16 Rate 2.5 Gain 0.7804





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